On the Connectivity in One-Dimensional Ad Hoc Wireless Networks with a Forbidden Zone

Xiaodong Hu[1], Evgeniy Grechnikov[2]

[1] Department of Mathematics, Shandong University, Jinan 250100, China.

[2] Department of Physics and Mathematics, Bauman Moscow State Technical University, Moscow 47076, Russia.

Abstract

This paper investigates the connectivity in one-dimensional ad hoc wireless networks with a forbidden zone. We derive the probability of the wireless networks which are composed of exactly m clusters by means of the methods of combinatorics and probability. The probability of connectivity, i.e. $m = 1$, can be obtained as a special case. Further, we explain how the transmission range of node affects the connectivity of the wireless network.

Mathematics Subject Classification 2010 60C05, 05C40, 94C15

Key Words Combinatorial problems, interconnection networks, connectivity, a forbidden zone, cluster.

1 Introduction

In recent years, ad hoc wireless networks have been extensively investigated due to their wide applications in computer communication and engineering. A fundamental and important issue is the connectivity of ad hoc network which was introduced in [2, 9]. Many rigorous results on the asymptotic critical transmission radius and asymptotic critical neighbors for the connectivity of network in a bounded area have been obtained to date [1, 2, 4, 10]. More recently, the one-dimensional version which is also called the random interval graph has been studied distinguishingly in the literature [11, 12]. However, these ad hoc wireless networks are usually assumed in a regular graph, which contradict with the practical applications. We always need to consider the case of an irregular graph. For example, in the researches on monitoring the ocean temperature for an accurate weather prediction, detecting the

[1]Email: `xdhu@mspil.edu.cn`

[2]Email: `grechnikov@gmail.com`

forest fires occurring in remote area and rapid propagation of traffic information from vehicle to vehicle, we need to deal with the connectivity of ad hoc networks in an irregular region [7]. As each node in the ad hoc networks has a limit transmission power and the networks have to communicate in the way of multihop, we can not keep the ad hoc networks connected in general. This will lead to the difference of connectivity between the ad hoc network in a regular graph and an irregular one. Based on the purpose, we introduce a new one-dimensional ad hoc wireless network model with a forbidden zone and study its connectivity. We not only intend to investigate the effect that the transmission radius of nodes and the feature of irregular graph make, but also to make those engineers and theoretical computer scientists pay attention to the interesting topic.

The probability that a one-dimensional wireless ad hoc network which is composed of at most m clusters is presented in [4]. As a special case, the probability of network connectivity, *i.e.* $m = 1$ is also derived. In this paper, we obtain several generic formulas for the probability that a one-dimensional ad hoc network with a forbidden zone is composed of exactly m clusters. As some applications of the theorems, we analyze and confirm these results by plotting some figures. These results developed in this paper may be useful for the design and engineering of random wireless access networks in the future.

2. Main results and proofs

In order to state our main results, we first introduce some definitions. Let $G^{[a,b]}(n, L + b - a)$ denote the one-dimensional ad hoc wireless network with n nodes and a forbidden zone $[a, b]$, such that $0 \le a < b \le L$. In general, assume that all nodes in the ad hoc networks are independently and identically distributed (*i.i.d.*) in the two-closed interval $[0, a] \cup [b, L + b - a]$, and a pair of nodes can directly communicate if and only if the distance between them is not larger than their transmission radius r $(0 < r < L)$. Obviously, the case of $a = 0$ or $b = L$ is equivalent to the classical one-dimensional wireless ad hoc. So we always assume that $0 < a < \frac{L}{2}$ in virtue of symmetry. Let $G(n, L)$ denote the one-dimensional ad hoc network without a forbidden zone and with n nodes which are *i.i.d.* in $[0, L]$. The model is also called random interval graph which was investigated in [5] and [8]. Further, if there exists another fixed node at the point $x \in [0, L]$ (*i.e.* there are $n+1$ nodes in the interval $[0, L]$), denote the model by $G_x(n, L)$. In particular, when $x = 0$, we simply denote $G_0(n, L)$. From [11], the exact probability of connectivity in $G_x(n, L)$ has been derived. For simplicity, we also use $P_m(\cdot)$ to express the probability that the above wireless network is composed of at most m clusters and $Q_m(\cdot)$ for exactly m clusters. Easily see that $P_1(\cdot) \equiv Q_1(\cdot)$. The following results are known to us and will be needed later.

Lemma A. (see [3,6].)

$$Q_1\big(G(n, L)\big) = \sum_{i=0}^{k_1}(-1)^i C_{n-1}^i (1 - \frac{ir}{L})^n \tag{1}$$

and

$$Q_1\big(G_0(n, L)\big) = \sum_{i=0}^{k_2}(-1)^i C_n^i (1 - \frac{ir}{L})^n, \tag{2}$$

where $k_1 = \min\{n-1, \lfloor \frac{L}{r} \rfloor\}$, $k_2 = \min\{n, \lfloor \frac{L}{r} \rfloor\}$ and $\lfloor x \rfloor$ denotes the greatest integer which is not bigger than x. In general, the following theorem also holds.

Lemma B. (see [11].) For any $x \in [0.L]$, then

$$Q_1\big(G_x(n, L)\big) = \sum_{i=0}^{n} C_n^i (\frac{x}{L})^i (1 - \frac{x}{L})^{n-i} Q_1\big(G_0(i, x)\big) Q_1\big(G_0(n - x, L - x)\big), \tag{3}$$

where $Q_1\big(G_0(i, x)\big)$ is the expression of (2).

Lemma C. (see [4].) Let m be any natural number, then

$$P_m\big(G(n, L)\big) = 1 - \sum_{i=m}^{k_1}(-1)^{i-m} C_{i-1}^{m-1} C_{n-1}^i (1 - \frac{ir}{L})^n, \tag{4}$$

where $k_1 = \min\{n - 1, \lfloor \frac{L}{r} \rfloor\}$.

Now let's consider the connectivity of one-dimensional ad hoc networks with a single access point and a forbidden zone, respectively. The following four theorems are our main results.

Theorem 1. Let m be any natural number, then

$$Q_m\big(G(n, L)\big) = \sum_{i=m-1}^{k_1} (-1)^{i-m+1} C_i^{m-1} C_{n-1}^i (1 - \frac{ir}{L})^n \tag{5}$$

and

$$Q_m\big(G_0(n, L)\big) = \sum_{i=m-1}^{k_2} (-1)^{i-m+1} C_i^{m-1} C_n^i (1 - \frac{ir}{L})^n, \tag{6}$$

where $k_1 = \min\{n - 1, \lfloor \frac{L}{r} \rfloor\}$, $k_2 = \min\{n, \lfloor \frac{L}{r} \rfloor\}$ and $\sum_{i=m}^{k}(\cdot) := 0$ for $k < m$.

Proof. First we come to consider (6). The proof comes from [4]. In order to avoid boring the reader, we shall not prove it in detail and only give the different part.

Let $\{v_i\}_{i=0}^n$ represent respectively the positions of $n+1$ nodes, where $v_0 = 0$. From the definition of the model, $\{v_i\}_{i=1}^n$ are *i.i.d.*. Order these random variables, we easily obtain $0 = v_{(0)} \le v_{(1)} \le \ldots \le v_{(n)} \le L$. Define $\Delta_{i-1} = v_{(i)} - v_{(i-1)}, v_0 = 0$. we have

$$0 \le \Delta_{(0)} \le \Delta_{(1)} \le \ldots \le \Delta_{(n-1)},$$

where $\{\Delta_{(i)}\}_{0\le i\le n-1}$ denote the order statistics of random variables $\{\Delta_i\}_{0\le i\le n-1}$.

For any k constants $c_i \ge 0$, $i = 0, 1, \ldots, k-1$, satisfying $\sum_{i=0}^{k-1} c_i \le L$, applying the properties of order statistics, we have

$$P\big(\Delta_0 > c_0, \ldots, \Delta_{k-1} > c_{k-1}\big) = \frac{(L - c_0 - \ldots - c_{k-1})^n}{L^n}.$$

Note that the probability that the network is composed of at most m clusters is $P(\Delta_{(n-m)} \le r)$ and the joint distribution for any k of the Δ_i ($k = 0, \cdots, n-1$) is the same as that of the first k. We further have

$$\begin{aligned} P(\Delta_{(n-m)} > r) &= \sum_{i=m}^{n} (-1)^{i-m} C_{i-1}^{m-1} C_n^i P(\Delta_0 > r, \ldots, \Delta_{m-1} > r) \\ &= \sum_{i=m}^{k_2} (-1)^{i-m} C_{i-1}^{m-1} C_n^i (1 - \frac{ir}{L})^n, \end{aligned}$$

where $k_2 = \min\{n, \lfloor \frac{L}{r} \rfloor\}$.

Thus, $P_m(G_0(n,L)) = 1 - \sum_{i=m}^{k_2} (-1)^{i-m} C_{i-1}^{m-1} C_n^i (1 - \frac{ir}{L})^n$ and

$$\begin{aligned} Q_m(G_0(n,L)) &= P_m(G_0(n,L)) - P_{m-1}(G_0(n,L)) \\ &= \sum_{i=m-1}^{k_2} (-1)^{i-m+1} C_{i-1}^{m-2} C_n^i (1 - \frac{ir}{L})^n - \sum_{i=m}^{k_2} (-1)^{i-m} C_{i-1}^{m-1} C_n^i (1 - \frac{ir}{L})^n \\ &= \sum_{i=m-1}^{k_2} (-1)^{i-m+1} C_i^{m-1} C_n^i (1 - \frac{ir}{L})^n. \end{aligned}$$

On the other hand, we can obtain (5) by using the same method, *i.e.*

$$\begin{aligned} Q_m(G(n,L)) &= P_m(G(n,L)) - P_{m-1}(G(n,L)) \\ &= \sum_{i=m-1}^{k_1} (-1)^{i-m+1} C_{i-1}^{m-2} C_{n-1}^i (1 - \frac{ir}{L})^n - \sum_{i=m}^{k_1} (-1)^{i-m} C_{i-1}^{m-1} C_{n-1}^i (1 - \frac{ir}{L})^n \\ &= \sum_{i=m-1}^{k_1} (-1)^{i-m+1} C_i^{m-1} C_{n-1}^i (1 - \frac{ir}{L})^n. \end{aligned}$$

Remark 1. Let $m=1$, we obtain the exact probability that the wireless network with a single access node at point 0 is connected, *i.e.*

$$P_1\big(G_0(n,L)\big)=Q_1\big(G_0(n,L)\big)=\sum_{i=0}^{k_2}(-1)^iC_n^i(1-\frac{ir}{L})^n.$$

This is exactly the expression of (2).

If the single access point is not placed at the point 0 or L, then we have the following theorem.

Theorem 2. Let m be any natural number and $x\in(0,L)$, then

$$\begin{aligned}Q_m\big(G_x(n,L)\big)&=Q_m\big(G_0(n,x)\big)(\frac{x}{L})^n+Q_m\big(G_0(n,L-x)\big)(\frac{L-x}{L})^n\\&+\sum_{i=1}^{n-1}\sum_{j=k_3}^{k_4}C_n^iQ_j\big(G_0(i,x)\big)Q_{m+1-j}\big(G_0(n-i,L-x)\big)(\frac{x}{L})^i(\frac{L-x}{L})^{n-i},\quad(6)\end{aligned}$$

where $k_3=\max\{1,m+i-n\}$, $k_4=\min\{i+1,m\}$.

Proof. Applying the total probability formula, we have

$$Q_m\big(G_x(n,L)\big)=\sum_{i=0}^{n}Q_m\big(G_x(n,L)\big|i\text{ nodes in }[0,x]\big)P\big(i\text{ nodes in }[0,x]\big).\qquad(7)$$

Further,

$$P\big(i\text{ nodes in }[0,x]\big)=C_n^i(\frac{x}{L})^i(\frac{L-x}{L})^{n-i}$$

and

$$\begin{aligned}Q_m\big(G_x(n,L)\big|i\text{ nodes in }[0,x]\big)&=\sum_{j=k_3}^{k_4}Q_j\big(G_0(i,x)\big)Q_{m+1-j}\big(G_0(n-i,L-x)\big)\\&+Q_m\big(G_0(i,x)\big)+Q_m\big(G_0(n-i,L-x)\big),\qquad(8)\end{aligned}$$

where $k_3=\max\{1,(m+i-n)\}$ and $k_4=\min\{i+1,m\}$.

Indeed, if there are $i+1$ nodes, including the fixed node at x, placed in $[0,x]$, the event that there are only m clusters in this wireless network is equivalent to the collection of events that j clusters in $[0,x]$ and $m+1-j$ clusters in $[x,L]$. According to the model, the restrictions on j are

$$1\le j\le i+1,\ j\le m\text{ and }\ 1\le m+1-j\le n+1-i.$$

That is

$$\max\{1,m+i-n\}\le j\le\{i+1,m\}.$$

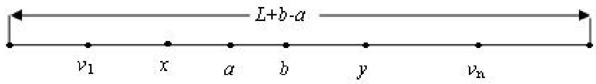

FIGURE 1

So we obtain (8). Finally, from (7), we have

$$\begin{aligned} Q_m\big(G_x(n,L)\big) &= Q_m\big(G_0(n,x)\big)(\frac{x}{L})^n + Q_m\big(G_0(n,L-x)\big)(\frac{L-x}{L})^n \\ &+\sum_{i=1}^{n-1}\sum_{j=k_3}^{k_4} C_n^i(\frac{x}{L})^i(\frac{L-x}{L})^{n-i}Q_j\big(G_0(i,x)\big)Q_{m+1-j}\big(G_0(n-i,L-x)\big) \\ &= \sum_{i=0}^{n}\sum_{j=k_3}^{k_4} C_n^i(\frac{x}{L})^i(\frac{L-x}{L})^{n-i}Q_j\big(G_0(i,x)\big)Q_{m+1-j}\big(G_0(n-i,L-x)\big), \end{aligned}$$

where $k_3 = \max\{1,(m+i-n)\}$, $k_4 = \min\{i+1,m\}$ and $Q_k\big(G_0(0,l)\big) := 1$ for any l and natural number k.

In particular, $m = 1$,

$$Q_1\big(G_x(n,L)\big) = \sum_{i=0}^{n} C_n^i(\frac{x}{L})^i(\frac{L-x}{L})^{n-i}Q_1\big(G_0(i,x)\big)Q_1\big(G_0(n-i,L-x)\big). \quad (9)$$

The proof is completed.

Next, we shall analyze the exact probability that a one-dimensional ad hoc network with a forbidden zone $[a,b]$ is composed of exactly m clusters. As a special case, the connectivity is also considered.

We still assume that the n nodes can be placed on the intervals $[0,a]$ or $[b,L+b-a]$, but not in (a,b). A clear intuition shows that the network consists of two parts under the condition $b-a > r$. (See FIGURE 1) From the knowledge on probability, we can see that the probability that only two nodes in the network placed respectively at the points a and b is zero. So we have the following result.

Theorem 3. If $b-a \geq r$, then for any natural number $m \geq 2$,

$$\begin{aligned} Q_m((G^{[a,b]}(n,L)) &= \sum_{i=1}^{n-1}\sum_{j=k_5}^{k_6} C_n^i(\frac{a}{L})^i(\frac{L-a}{L})^{n-i}Q_j(G(i,a))Q_{m-j}(G(n-i,L-a)) \\ &+(\frac{a}{L})^nQ_m(G(n,a)) + (\frac{L-a}{L})^nQ_m(G(n,L-a)), \quad (10) \end{aligned}$$

where $k_5 = \max\{m+i-n,1\}$ and $k_6 = \min\{i,m-1\}$.

In particular, $m=2$,

$$Q_2\big(G^{[a,b]}(n,L)\big)=(\frac{a}{L})^nQ_2\big(G(n,a)\big)+(\frac{L-a}{L})^nQ_2\big(G(n,L-a)\big)$$
$$+\sum_{i=1}^{n-1}C_n^iQ_1\big(G(i,a)\big)Q_1\big(G(n-i,L-a)\big)(\frac{a}{L})^i(\frac{L-a}{L})^{n-i}. \quad (11)$$

Proof. Applying the total probability formula, we have

$$Q_m(G^{[a,b]}(n,L))=\sum_{i=0}^{n}Q_m\big(G^{[a,b]}(n,L)\big|i \text{ nodes in } [0,a]\big)P(i \text{ nodes in } [0,a]). \quad (12)$$

Further,

$$P(i \text{ nodes in } [0,a])=C_n^i(\frac{a}{L})^i(\frac{L-a}{L})^{n-i} \quad (13)$$

and for $i\geq 1$,

$$Q_m\big(G^{[a,b]}(n,L)\big|i \text{ nodes in } [0,a]\big)=\sum_{j=k_5}^{k_6}Q_j(G(i,a))Q_{m+1-j}(G(n-i,L-a)). \quad (14)$$

where $k_5=\max\{m+i-n,1\}$ and $k_6=\min\{i,m-1\}$.

Replace (12) by (13) and (14), we easily obtain (10).

When $b-a<r$, the generic formula for the probability that a wireless network with a forbidden zone $[a,b]$ consists of exactly m clusters is complicated.

Theorem 4. If $b-a<r$, then for any natural number $m\geq 1$,

$$Q_m(G^{[a,b]}(n,L))=(\frac{a}{L})^nQ_m(G(n,a))+(\frac{L-a}{L})^nQ_m(G(n,L-a))$$
$$+\frac{n(n-1)}{L^n}\sum_{i=0}^{n-2}\sum_{j=k_7}^{k_8}\int_0^a\Big[\int_{A_1}^{L+b-a}C_{n-2}^ix^i(L+b-a-y)^{n-2-i}$$
$$\times Q_j(G_0(i,x))Q_{m-j}(G_0(n-2-i,L+b-a-y))dy\Big]dx$$
$$+\frac{n(n-1)}{L^n}\sum_{i=0}^{n-2}\sum_{j=k_9}^{k_{10}}\int_{A_2}^a\Big[\int_b^{A_3}C_{n-2}^ix^i(L+b-a-y)^{n-2-i}$$
$$\times Q_j(G_0(i,x))Q_{m+1-j}(G_0(n-2-i,L+b-a-y))dy\Big]dx,$$

where $k_7=\max\{1,m+1+i-n\}$, $k_8=\min\{m-1,i+1\}$, $k_9=\max\{1,m+i+2-n\}$, $k_{10}=\min\{m,i+1\}$, $A_1=\min\{L+b-a,\max\{x+r,b\}\}$, $A_2=\max\{0,b-r\}$ and $A_3=\min\{L+b-a,x+r\}$.

In particular, if $m = 1$, $a \le \frac{L}{2}$ and $b - a < r < b$,

$$Q_1(G^{[a,b]}(n, L)) = (\frac{a}{L})^n Q_1(G(n, a)) + (\frac{L-a}{L})^n Q_1(G(n, L-a))$$
$$+\frac{n(n-1)}{L^n}\sum_{i=0}^{n-2} C_{n-2}^i \int_{b-r}^{a} \Big[\int_b^{x+r} x^i(L+b-a-y)^{n-2-i}$$
$$\times Q_1(G_0(i, x))Q_1(G_0(n-2-i, L+b-a-y))dy\Big]dx. \qquad (15)$$

Proof. First consider the case of $m = 1$. To keep the wireless network connected, only three possibilities of events should be considered. The former two possibilities are that n nodes consisting of one cluster are uniformly placed in $[0, a]$ or $[b, L+b-a]$. Obviously, the sum of the two probabilities is

$$(\frac{a}{L})^n P_1(G(n, a)) + (\frac{L-a}{L})^n P_1(G(n, L-a)). \qquad (16)$$

The third event is that i $(1 \le i \le n)$ nodes are placed in $[0, a]$, $n - i$ nodes in $[b, L + b - a]$ and the whole nodes consists of exactly one cluster. To compute its probability, we choose any two nodes from n nodes of the network and then place them respectively at x, y, where $x \in [0, a]$ and $y \in [b, L + b - a]$. We assume that there are no other nodes between x and y, *i.e.* the left $n - 2$ nodes are placed in $[0, x]$ and $[y, L + b - a]$. Let the n nodes consist of one cluster. Easily find the number of total choices is $n(n - 1)$ and the restrictive conditions for x and y are

$$0 \le x \le b - r \quad \text{and} \quad b \le y \le x + r.$$

So the probability of the third event is equivalent to

$$C_{n-2}^i \frac{n(n-1)}{L^n}\sum_{i=0}^{n-2} \int_{b-r}^{a} \Big[\int_b^{x+r} x^i(L+b-a-y)^{n-2-i}$$
$$\times Q_1\big(G_0(i, x)\big)Q_1\big(G_0(n-2-i, L+b-a-y)\big)dy\Big]dx, \qquad (17)$$

by applying total probability formula. From (16) and (17), (15) can be proved at once.

The following step is to prove the former part of Theorem 4.

Using the same method as the case of $m = 1$, we still have three possible events that the wireless network is composed of exactly m $(m \ge 2)$ clusters. The sum of the former two probabilities is

$$(\frac{a}{L})^n Q_m(G(n, a)) + (\frac{L-a}{L})^n Q_m(G(n, L-a)). \qquad (18)$$

Now come to the third event. The computation is more complex. We still use the former method. Choose any two nodes from n nodes of the network and place them respectively at the points x, y. According to the model and noting that $m \geq 2$, we need to consider the two cases of $|x-y| \leq r$ or $|x-y| > r$, *i.e.* the two nodes are connected or not. For each case, we also have $n(n-1)$ ways to choose. When $|x-y| \leq r$, we need to assume that

$$b - r \leq x \leq a \qquad \text{and} \qquad b \leq y \leq x + r.$$

Furthermore, these nodes in $[0, x]$ consist of only j $(j = 1, \cdots, m-1)$ clusters and those in $[y, L+b-a]$, $m+1-j$ clusters. By the total probability formula, we have

$$P(\text{the third event}) = \frac{n(n-1)}{L^n} \sum_{i=0}^{n-2} \sum_{j=k_9}^{k_{10}} \int_{A_2}^{a} \Big[\int_{b}^{A_3} C_{n-2}^{i} x^i (L+b-a-y)^{n-2-i} \times Q_j(G_0(i,x)) Q_{m+1-j}(G_0(n-2-i, L+b-a-y)) dy \Big] dx, \quad (19)$$

where $k_9 = \max\{1, m+i+2-n\}$ and $k_{10} = \min\{m, i+1\}$, $A_2 = \max\{0, b-r\}$ and $A_3 = \min\{L+b-a, x+r\}$.

When $|x-y| > r$, we need to assume that

$$0 \leq x \leq a \qquad \text{and} \qquad \max\{b, x+r\} \leq y \leq L+b-a.$$

Then there are only j clusters in $[0, x]$ and $m-j$ in $[y, l+b-a]$. So we similarly have

$$P(\text{the third event}) = \frac{n(n-1)}{L^n} \sum_{i=0}^{n-2} \sum_{j=k_7}^{k_8} \int_{0}^{a} \Big[\int_{A_1}^{L+b-a} C_{n-2}^{i} x^i (L+b-a-y)^{n-2-i} \times Q_j(G_0(i,x)) Q_{m-j}(G_0(n-2-i, L+b-a-y)) dy \Big] dx, \quad (20)$$

where $k_7 = \max\{1, m+1+i-n\}$, $k_8 = \min\{m-1, i+1\}$ and $A_1 = \min\{L+b-a, \max\{x+r, b\}\}$. From (18), (19) and (20), we easily obtain the expression of $P_m(G^{[a,b]}(n, L))$.

3. Discussion

In FIGURE 2, we plot $Q_m(G_x(n, L))$ as a function of the normalized range $\frac{r}{L}$ for different values of m, n and x. From (a) and (b), we can observe that the networks almost can't be composed of exactly m $(m \geq 2)$ clusters once the value of $\frac{r}{L}$ becomes smaller than 0.05 or larger than 0.2. The maximum value for $Q_m(G_0(n, L))$ becomes much bigger as n becomes smaller at the fixed point $x = 0$, but becomes smaller as m becomes bigger. In virtue of (c), we can see that the properties of $Q_m(G_x(n, L))$ is similar to ones of $Q_m(G_0(n, L))$ for each m and n. We avidly guess that the

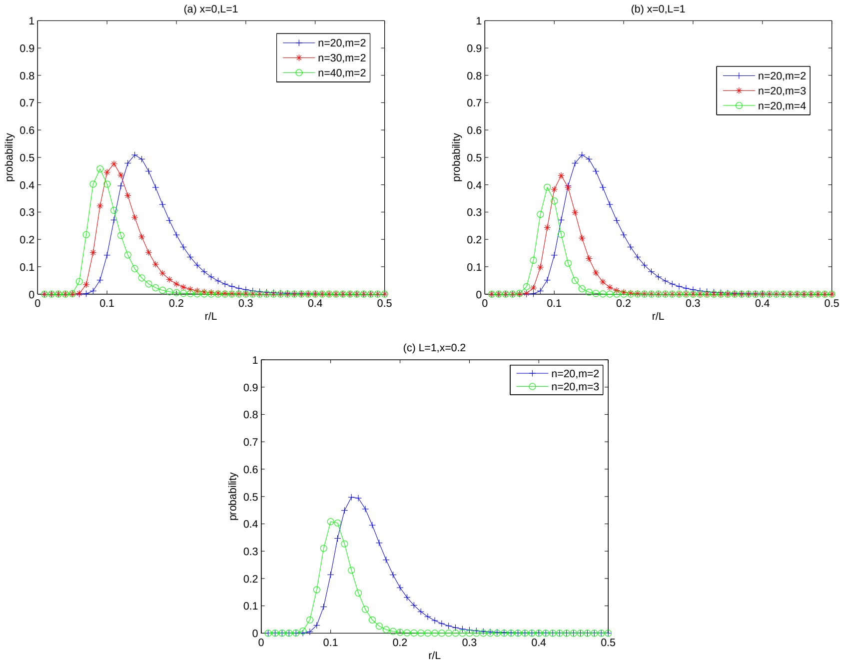

FIGURE 2

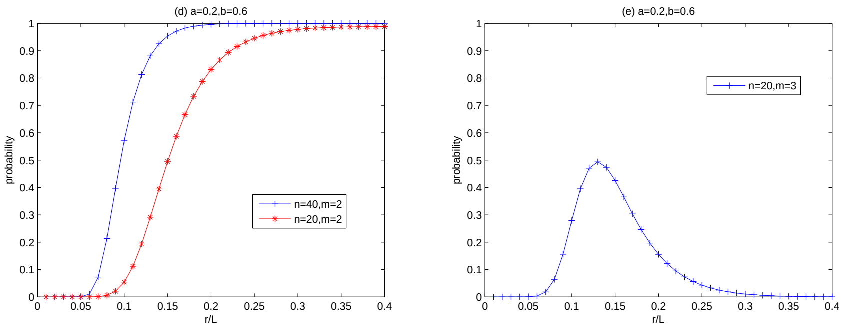

FIGURE 3

maximum value of $P_m(G_x(n,L))$ is no more than 0.5. In fact, we can prove the result with the help of mathematical software such as Maple 6.0.

In FIGURE 3, we plot $P_m(G^{[a,b]}(n,L))$ as a function of normalized range $\frac{r}{L}$ for different values of a, b which satisfy $b - a > r$. From (d), we also observe that the value of $P_m(G^{[a,b]}(n,L))$ becomes more large as $\frac{r}{L}$ increases for $a = 0.2$ and $b = 0.6$. When $\frac{r}{L} \leq 0.05$, the network can not be composed of only two clusters. Once increase n, the network becomes more connected under fixed a and b. An interesting problem is what conditions can guarantee that $P_m(G^{[a,b]}(n,L))$ can take its maximum for each forbidden zone as the length $b - a$ is fixed. Finally, we have not a good method to deal with the results of Theorem 4 at present. Especially, we want to know how much effect the transmission range r makes for the network. We believe the problems can be solved by approximation methods. This is our future work.

References

[1] Antunes N., Jacinto G. and Pacheco A. On the minimum hop count and connectivity in one-dimensional ad hoc wireless networks. Telecom. Syst. 39 (2008) 137-143.

[2] Cheng Y. C. and Robertazzi T. G. Critical connectivity phenomena in multihop radio models. IEEE Trans. Commu. 37 (1989) 770-777.

[3] Desai M. and Manjunath D. On the connectivity in finite ad hoc networks. IEEE Commun. Lett. 6(10) (2002) 437-439.

[4] Ghasemi A. and Nader-Esfahnai S. Exact probability of cononectivity in one-dimensional ad hoc wireless networks. IEEE Commun. Lett. 10(4) (2006) 251-263.

[5] Godehardt and Jaworski J. On the connecivity of a random interval graph. Rand. Struct. Algo. 9(1) (1996) 137-161.

[6] Gore A.D. Comments on the connectivity in finite ad hoc networks. IEEE Commun. Lett. 10(2) (2006) 88-90.

[7] Kumar A. and Manjunath D. A tutorial survey of topics in wireless networking: Part II, Sādhanā 32(6) (2007) 645-681.

[8] Penrose M. D. Random Geometric Graphs. Oxford University Press, Oxford, 2003.

[9] Philips T. K., Pandwar S. S. and Tantawi A. N. Connectivity properties of a packet radio network model. IEEE Trans Inf. Theory 35 (1989) 1044-1047.

[10] Santi P. and Blough D. M. The critical transmitting range for connectivity in sparse wireless ad hoc networks. IEEE Trans. Mobile Comput. 2(1) (2003) 25-39.

[11] Shang Y. L. Connectivity in a random interval graph with access points. Infor. Proc. Lett. 109(9) (2009) 446-449.

[12] Wang H. X., Lu G. L., Jia W. J. and Zhao W. Connectivity in finite ad hoc networks. Science in China, Series F 51(4) (2008) 417-424.