P, C and T for Truly Neutral Particles

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Abstract. We study Majorana-like constructs in the $(1/2,0)\oplus(0,1/2)$ representation. We show that the parity properties of a fermion and its antifermion are quite different comparing with the Dirac case. The transformation laws for C and T operations have also been given on the secondary quantization level. The construct can be applied to explanation of the present situation in neutrino physics. The case of the $(1,0)\oplus(0,1)$ field is also considered.

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During the 20th century various authors introduced *self/anti-self* charge-conjugate 4-spinors (including in the momentum representation), see [1, 2, 3, 4]. Later, Lounesto, Dvoeglazov, Kirchbach *etc*, Ref. [5, 6, 7, 8] studied these spinors, they found dynamical equations, gauge transformations and other specific features of them. Recently, in [8] it was claimed that "for imaginary C parities, the neutrino mass can drop out from the single β decay trace and reappear in $0\nu\beta\beta$,... in principle experimentally testable signature for a non-trivial impact of Majorana framework in experiments with polarized sources" (see also Summary of the cited paper). Thus, phase factors can have physical significance in quantum mechanics. The aim of my talk is to remind that several researchers presented in the 90s concerning with the neutrino description.

The definitions are:

$$C = e^{i\theta_c}\begin{pmatrix} 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & i & 0 & 0\\ -i & 0 & 0 & 0\end{pmatrix}\mathcal{K} = -e^{i\theta_c}\gamma^2\mathcal{K} \tag{1}$$

is the anti-linear operator of charge conjugation. We define the *self/anti-self* charge-conjugate 4-spinors in the momentum space[1]

$$C\lambda^{S,A}(p^\mu) = \pm\lambda^{S,A}(p^\mu)\,,\quad C\rho^{S,A}(p^\mu) = \pm\rho^{S,A}(p^\mu)\,, \tag{2}$$

where

$$\lambda^{S,A}(p^\mu) = \begin{pmatrix} \pm i\Theta\phi_L^*(p^\mu)\\ \phi_L(p^\mu)\end{pmatrix}\,,\quad \rho^{S,A}(p^\mu) = \begin{pmatrix} \phi_R(p^\mu)\\ \mp i\Theta\phi_R^*(p^\mu)\end{pmatrix}\,. \tag{3}$$

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[1] In [8] a bit different notation was used referring to [2].

The Wigner matrix is $\Theta_{[1/2]} = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, and ϕ_L, ϕ_R are the Ryder (Weyl) left- and right-handed 2-spinors

$$\begin{aligned}
\phi_R(p^\mu) &= \Lambda_R(\mathbf{p} \leftarrow \mathbf{0})\phi_R(\mathbf{0}) = \exp(+\sigma \cdot \varphi/2)\phi_R(\mathbf{0})\,, && (4)\\
\phi_L(p^\mu) &= \Lambda_L \mathbf{p} \leftarrow \mathbf{0})\phi_L(\mathbf{0}) = \exp(-\sigma \cdot \varphi/2)\phi_L(\mathbf{0})\,, && (5)
\end{aligned}$$

with $\varphi = \mathbf{n}\varphi$ being the boost parameters:

$$cosh\varphi = \gamma = \frac{1}{\sqrt{1-v^2/c^2}}\,, sinh\varphi = \beta\gamma = \frac{v/c}{\sqrt{1-v^2/c^2}}\,, tanh\varphi = v/c\,. \qquad (6)$$

As we have shown the 4-spinors λ and ρ are NOT the eigenspinors of helicity (cf. [9]). Moreover, λ and ρ are NOT the eigenspinors of the parity $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} R$, as opposed to the Dirac case.

Such definitions of 4-spinors differ, of course, from the original Majorana definition in x-representation:

$$\nu(x) = \frac{1}{\sqrt{2}}(\Psi_D(x) + \Psi_D^c(x))\,, \quad a_\sigma(\mathbf{p}) = \frac{1}{\sqrt{2}}(b_\sigma(\mathbf{p}) + d_\sigma^\dagger(\mathbf{p}))\,, \qquad (7)$$

$$\nu(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} \sum_\sigma [u_\sigma(\mathbf{p}) a_\sigma(\mathbf{p}) e^{-ip\cdot x} + v_\sigma(\mathbf{p})[\lambda a_\sigma^\dagger(\mathbf{p})] e^{+ip\cdot x}]\,, \qquad (8)$$

$C\nu(x) = \nu(x)$, that represents the positive real $C-$ parity field operator. However, the momentum-space Majorana-like spinors open various possibilities for description of neutral particles (with experimental consequences, see [8]).

The 4-spinors of the second kind $\lambda_{\uparrow\downarrow}^{S,A}(p^\mu)$ and $\rho_{\uparrow\downarrow}^{S,A}(p^\mu)$ are [7]:

$$\lambda_\uparrow^S(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} ip_l \\ i(p^- + m) \\ p^- + m \\ -p_r \end{pmatrix}, \lambda_\downarrow^S(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} -i(p^+ + m) \\ -ip_r \\ -p_l \\ (p^+ + m) \end{pmatrix}, \qquad (9)$$

$$\lambda_\uparrow^A(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} -ip_l \\ -i(p^- + m) \\ (p^- + m) \\ -p_r \end{pmatrix}, \lambda_\downarrow^A(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} i(p^+ + m) \\ ip_r \\ -p_l \\ (p^+ + m) \end{pmatrix}, \qquad (10)$$

$$\rho_\uparrow^S(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} p^+ + m \\ p_r \\ ip_l \\ -i(p^+ + m) \end{pmatrix}, \rho_\downarrow^S(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} p_l \\ (p^- + m) \\ i(p^- + m) \\ -ip_r \end{pmatrix}, \qquad (11)$$

$$\rho_\uparrow^A(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} p^+ + m \\ p_r \\ -ip_l \\ i(p^+ + m) \end{pmatrix}, \rho_\downarrow^A(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} p_l \\ (p^- + m) \\ -i(p^- + m) \\ ip_r \end{pmatrix}. \qquad (12)$$

with $p_r = p_x + ip_y$, $p_l = p_x - ip_y$, $p^{\pm} = p_0 \pm p_z$. The indices $\uparrow\downarrow$ should be referred to the "chiral helicity" quantum number introduced in the 60s, $\eta = -\gamma^5 \hat{S}_3$. While

$$Pu_\sigma(\mathbf{p}) = +u_\sigma(\mathbf{p})\,, Pv_\sigma(\mathbf{p}) = -v_\sigma(\mathbf{p})\,, \tag{13}$$

we have

$$P\lambda^{S,A}(\mathbf{p}) = \rho^{A,S}(\mathbf{p})\,, P\rho^{S,A}(\mathbf{p}) = \lambda^{A,S}(\mathbf{p})\,, \tag{14}$$

for the Majorana-like momentum-space 4-spinors on the first quantization level. In this basis one has

$$\rho^S_\uparrow(p^\mu) \;=\; -i\lambda^A_\downarrow(p^\mu)\,, \rho^S_\downarrow(p^\mu) = +i\lambda^A_\uparrow(p^\mu)\,, \tag{15}$$

$$\rho^A_\uparrow(p^\mu) \;=\; +i\lambda^S_\downarrow(p^\mu)\,, \rho^A_\downarrow(p^\mu) = -i\lambda^S_\uparrow(p^\mu)\,. \tag{16}$$

The normalization of the spinors $\lambda^{S,A}_{\uparrow\downarrow}(p^\mu)$ and $\rho^{S,A}_{\uparrow\downarrow}(p^\mu)$ are the following ones:

$$\overline{\lambda}^S_\uparrow(p^\mu)\lambda^S_\downarrow(p^\mu) \;=\; -im\,, \overline{\lambda}^S_\downarrow(p^\mu)\lambda^S_\uparrow(p^\mu) = +im\,, \tag{17}$$

$$\overline{\lambda}^A_\uparrow(p^\mu)\lambda^A_\downarrow(p^\mu) \;=\; +im\,, \overline{\lambda}^A_\downarrow(p^\mu)\lambda^A_\uparrow(p^\mu) = -im\,, \tag{18}$$

$$\overline{\rho}^S_\uparrow(p^\mu)\rho^S_\downarrow(p^\mu) \;=\; +im\,, \overline{\rho}^S_\downarrow(p^\mu)\rho^S_\uparrow(p^\mu) = -im\,, \tag{19}$$

$$\overline{\rho}^A_\uparrow(p^\mu)\rho^A_\downarrow(p^\mu) \;=\; -im\,, \overline{\rho}^A_\downarrow(p^\mu)\rho^A_\uparrow(p^\mu) = +im\,. \tag{20}$$

All other conditions are equal to zero. On the classical level in the generalized case $\bar{\lambda}^S_\uparrow \lambda^S_\downarrow = 2iN^2 \cos(\theta_1 + \theta_2)$.

First of all, one must derive dynamical equations for the Majorana-like spinors in order to see what dynamics do the neutral particles have. One can use the generalized form of the Ryder relation for zero-momentum spinors [10]:

$$\left[\phi^h_L(\mathbf{0})\right]^* = (-1)^{1/2-h}\, e^{-i(\vartheta^L_1+\vartheta^L_2)}\,\Theta_{[1/2]}\,\phi^{-h}_L(\mathbf{0})\,, \tag{21}$$

Relations for zero-momentum right spinors are obtained with the substitution $L \leftrightarrow R$. h is the helicity quantum number for the left- and right 2-spinors. Hence, implying that $\lambda^S(p^\mu)$ (and $\rho^A(p^\mu)$) answer for positive-frequency solutions; $\lambda^A(p^\mu)$ (and $\rho^S(p^\mu)$), for negative-frequency solutions, one can obtain the dynamical coordinate-space equations [6]

$$i\gamma^\mu\partial_\mu\lambda^S(x) - m\rho^A(x) \;=\; 0\,, \quad i\gamma^\mu\partial_\mu\rho^A(x) - m\lambda^S(x) = 0\,, \tag{22}$$

$$i\gamma^\mu\partial_\mu\lambda^A(x) + m\rho^S(x) \;=\; 0\,, \quad i\gamma^\mu\partial_\mu\rho^S(x) + m\lambda^A(x) = 0\,. \tag{23}$$

These are NOT the Dirac equations.[2] They can be written in the 8-component form as follows:

$$\left[i\Gamma^\mu\partial_\mu - m\right]\Psi_{(+)}(x) = 0\,, \quad \left[i\Gamma^\mu\partial_\mu + m\right]\Psi_{(-)}(x) = 0\,, \tag{24}$$

[2] Of course, the λ and ρ spinors obey also the Klein-Gordon-Fock equation, see my cited works.

with $\Psi_{(+)}(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix}$, $\Psi_{(-)}(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix}$, and $\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}$ Similar formulations have been presented by M. Markov [11] long ago, and A. Barut and G. Ziino [3]. The group-theoretical basis for such doubling has been first given in the papers by Gelfand, Tsetlin and Sokolik [12] and by other authors. Hence, the Lagrangian is

$$\mathscr{L} = \frac{i}{2}\Big[\bar{\lambda}^S\gamma^\mu\partial_\mu\lambda^S - (\partial_\mu\bar{\lambda}^S)\gamma^\mu\lambda^S + \bar{\rho}^A\gamma^\mu\partial_\mu\rho^A - (\partial_\mu\bar{\rho}^A)\gamma^\mu\rho^A +$$
$$\bar{\lambda}^A\gamma^\mu\partial_\mu\lambda^A - (\partial_\mu\bar{\lambda}^A)\gamma^\mu\lambda^A + \bar{\rho}^S\gamma^\mu\partial_\mu\rho^S - (\partial_\mu\bar{\rho}^S)\gamma^\mu\rho^S -$$
$$-m(\bar{\lambda}^S\rho^A + \bar{\rho}^A\lambda^S - \bar{\lambda}^A\rho^S - \bar{\rho}^S\lambda^A)\Big]\,. \tag{25}$$

The connection with the Dirac spinors has been found. For instance [4, 6],

$$\begin{pmatrix} \lambda^S_\uparrow(p^\mu) \\ \lambda^S_\downarrow(p^\mu) \\ \lambda^A_\uparrow(p^\mu) \\ \lambda^A_\downarrow(p^\mu) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & i & -1 & i \\ -i & 1 & -i & -1 \\ 1 & -i & -1 & -i \\ i & 1 & i & -1 \end{pmatrix}\begin{pmatrix} u_{+1/2}(p^\mu) \\ u_{-1/2}(p^\mu) \\ v_{+1/2}(p^\mu) \\ v_{-1/2}(p^\mu) \end{pmatrix}. \tag{26}$$

It was shown [6] that the covariant derivative (and, hence, the interaction) can be introduced in this construct in the following way:

$$\partial_\mu \to \nabla_\mu = \partial_\mu - ig\text{Ł}^5 B_\mu \quad , \tag{27}$$

where $\text{Ł}^5 = \mathrm{diag}(\gamma^5 \quad -\gamma^5)$, the 8×8 matrix. With respect to the transformations

$$\lambda'(x) \to (\cos\alpha - i\gamma^5\sin\alpha)\lambda(x), \quad \overline{\lambda}'(x) \to \overline{\lambda}(x)(\cos\alpha - i\gamma^5\sin\alpha) \quad , \tag{28}$$
$$\rho'(x) \to (\cos\alpha + i\gamma^5\sin\alpha)\rho(x), \quad \overline{\rho}'(x) \to \overline{\rho}(x)(\cos\alpha + i\gamma^5\sin\alpha) \tag{29}$$

the spinors retain their properties to be self/anti-self charge conjugate spinors and the proposed Lagrangian [6] remains to be invariant. This tells us that while self/anti-self charge conjugate states has zero eigenvalues of the ordinary (scalar) charge operator but they can possess the axial charge (cf. with the discussion of [3]).

In fact, from this consideration one can recover the Feynman-Gell-Mann equation (and its charge-conjugate equation). They are re-written in the two-component forms:

$$\begin{cases} \left[\pi^-_\mu\pi^{\mu\,-} - m^2 - \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right]\chi(x) = 0\,, \\ \left[\pi^+_\mu\pi^{\mu\,+} - m^2 + \frac{g}{2}\widetilde{\sigma}^{\mu\nu}F_{\mu\nu}\right]\phi(x) = 0\,, \end{cases} \tag{30}$$

where one now has $\pi^\pm_\mu = i\partial_\mu \pm gA_\mu$, $\sigma^{0i} = -\widetilde{\sigma}^{0i} = i\sigma^i$, $\sigma^{ij} = \widetilde{\sigma}^{ij} = \varepsilon_{ijk}\sigma^k$ and $v^{^{DL}}(x) = \mathrm{column}(\chi \quad \phi)$.

Next, because the transformations

$$\lambda'_S(p^\mu) = \begin{pmatrix} \Xi & 0 \\ 0 & \Xi \end{pmatrix}\lambda_S(p^\mu) \equiv \lambda^*_A(p^\mu), \lambda''_S(p^\mu) = \begin{pmatrix} i\Xi & 0 \\ 0 & -i\Xi \end{pmatrix}\lambda_S(p^\mu) \equiv -i\lambda^*_S, \tag{31}$$

$$\lambda'''_S(p^\mu) = \begin{pmatrix} 0 & i\Xi \\ i\Xi & 0 \end{pmatrix}\lambda_S(p^\mu) \equiv i\gamma^0\lambda^*_A(p^\mu), \lambda^{IV}_S(p^\mu) = \begin{pmatrix} 0 & \Xi \\ -\Xi & 0 \end{pmatrix}\lambda_S(p^\mu) \equiv \gamma^0\lambda^*_S, \tag{32}$$

with the 2×2 matrix Ξ defined as

$$\Xi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \quad , \quad \Xi\Lambda_{R,L}(\mathbf{0}\leftarrow\mathbf{p})\Xi^{-1} = \Lambda^*_{R,L}(\mathbf{0}\leftarrow\mathbf{p})\,, \tag{33}$$

and corresponding transformations for λ^A, do *not* change the properties of bispinors to be in the self/anti-self charge conjugate spaces, the Majorana-like field operator ($b^\dagger \equiv a^\dagger$) admits additional phase (and, in general, normalization) $SU(2)$ transformations:

$$\nu^{ML\,\prime}(x^\mu) = [c_0 + i(\tau\cdot\mathbf{c})]\,\nu^{ML\,\dagger}(x^\mu)\,, \tag{34}$$

where c_α are arbitrary parameters. The τ matrices are defined over the field of 2×2 matrices and the Hermitian conjugation operation is assumed to act on the c- numbers as the complex conjugation. One can parametrize $c_0 = \cos\phi$ and $\mathbf{c} = \mathbf{n}\sin\phi$ and, thus, define the $SU(2)$ group of phase transformations. One can select the Lagrangian which is composed from both field operators (with λ spinors and ρ spinors) and which remains to be invariant with respect to this kind of transformations. The conclusion is: a non-Abelian construct is permitted, which is based on the spinors of the Lorentz group only.

The Dirac-like and Majorana-like field operators can be built from both $\lambda^{S,A}(p^\mu)$ and $\rho^{S,A}(p^\mu)$, or their combinations. For instance,

$$\Psi(x^\mu) \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\sum_\eta \left[\lambda^S_\eta(p^\mu)\,a_\eta(\mathbf{p})\,\exp(-ip\cdot x) + \lambda^A_\eta(p^\mu)\,b^\dagger_\eta(\mathbf{p})\,\exp(+ip\cdot x)\right]. \tag{35}$$

The anticommutation relations are the following ones (due to the *bi-orthonormality*):[3]

$$[a_{\eta'}(p'^\mu), a^\dagger_\eta(p^\mu)]_\pm = (2\pi)^3 2E_p\delta(\mathbf{p}-\mathbf{p}')\delta_{\eta,-\eta'} \tag{36}$$

$$[b_{\eta'}(p'^\mu), b^\dagger_\eta(p^\mu)]_\pm = (2\pi)^3 2E_p\delta(\mathbf{p}-\mathbf{p}')\delta_{\eta,-\eta'} \tag{37}$$

Other (anti)commutators are equal to zero: ($[a_{\eta'}(p'^\mu), b^\dagger_\eta(p^\mu)] = 0$).

In the Fock space the operations of the charge conjugation and space inversions can be defined through unitary operators such that:

$$U^c_{[1/2]}\Psi(x^\mu)(U^c_{[1/2]})^{-1} = \mathscr{C}_{[1/2]}\Psi^\dagger_{[1/2]}(x^\mu), U^s_{[1/2]}\Psi(x^\mu)(U^s_{[1/2]})^{-1} = \gamma^0\Psi(x'^\mu), \tag{38}$$

the time reversal operation, through *an antiunitary* operator[4]

$$\left[V^T_{[1/2]}\Psi(x^\mu)(V^T_{[1/2]})^{-1}\right]^\dagger = S(T)\Psi^\dagger(x''^\mu)\,, \tag{39}$$

---

[3] It was noted the possibility of the generalization of the concept of the Fock space, which leads to the "doubling" Fock space [12, 3].

[4] Let us remind that the operator of hermitian conjugation does not act on c-numbers on the left side of the equation (39). This fact is conected with the properties of the antiunitary operator: $\left[V^T\lambda A(V^T)^{-1}\right]^\dagger = \left[\lambda^* V^T A(V^T)^{-1}\right]^\dagger = \lambda\left[V^T A^\dagger(V^T)^{-1}\right]$.

with $x'^{\mu} \equiv (x^0, -\mathbf{x})$ and $x''^{\mu} = (-x^0, \mathbf{x})$. We further assume the vacuum state to be assigned an even P- and C-eigenvalue and, then, proceed as in ref. [13]. As a result we have the following properties of creation (annihilation) operators in the Fock space:

$$
\begin{aligned}
U^s_{[1/2]} a_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} &= -i a_\downarrow(-\mathbf{p})\, , U^s_{[1/2]} a_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i a_\uparrow(-\mathbf{p})\, , \quad (40)\\
U^s_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} &= +i b^\dagger_\downarrow(-\mathbf{p})\, , U^s_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i b_\uparrow(-\mathbf{p})\, , \quad (41)
\end{aligned}
$$

that signifies that the states created by the operators $a^\dagger(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ have very different properties with respect to the space inversion operation, comparing with Dirac states (the case was also regarded in [3]):

$$
\begin{aligned}
U^s_{[1/2]} |\mathbf{p}, \uparrow>^+ = +i|-\mathbf{p}, \downarrow>^+\, , U^s_{[1/2]} |\mathbf{p}, \uparrow>^- = +i|-\mathbf{p}, \downarrow>^- \quad (42)\\
U^s_{[1/2]} |\mathbf{p}, \downarrow>^+ = -i|-\mathbf{p}, \uparrow>^+\, , U^s_{[1/2]} |\mathbf{p}, \downarrow>^- = -i|-\mathbf{p}, \uparrow>^- \quad (43)
\end{aligned}
$$

For the charge conjugation operation in the Fock space we have two physically different possibilities. The first one, *e.g.*,

$$
\begin{aligned}
U^c_{[1/2]} a_\uparrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\uparrow(\mathbf{p})\, , U^c_{[1/2]} a_\downarrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\downarrow(\mathbf{p}), \quad (44)\\
U^c_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\uparrow(\mathbf{p})\, , U^c_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\downarrow(\mathbf{p}), \quad (45)
\end{aligned}
$$

in fact, has some similarities with the Dirac construct. The action of this operator on the physical states are

$$
\begin{aligned}
U^c_{[1/2]} |\mathbf{p}, \uparrow>^+ &= +|\mathbf{p}, \uparrow>^-\, , U^c_{[1/2]} |\mathbf{p}, \downarrow>^+ = +|\mathbf{p}, \downarrow>^-\, , \quad (46)\\
U^c_{[1/2]} |\mathbf{p}, \uparrow>^- &= -|\mathbf{p}, \uparrow>^+\, , U^c_{[1/2]} |\mathbf{p}, \downarrow>^- = -|\mathbf{p}, \downarrow>^+\, . \quad (47)
\end{aligned}
$$

But, one can also construct the charge conjugation operator in the Fock space which acts, *e.g.*, in the following manner:

$$
\begin{aligned}
\widetilde{U}^c_{[1/2]} a_\uparrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = -b_\downarrow(\mathbf{p})\, , \widetilde{U}^c_{[1/2]} a_\downarrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = -b_\uparrow(\mathbf{p}), \quad (48)\\
\widetilde{U}^c_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\downarrow(\mathbf{p})\, , \widetilde{U}^c_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\uparrow(\mathbf{p}), \quad (49)
\end{aligned}
$$

and, therefore,

$$
\begin{aligned}
\widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^+ &= -|\mathbf{p}, \downarrow>^-\, , \widetilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow>^+ = -|\mathbf{p}, \uparrow>^-\, , \quad (50)\\
\widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^- &= +|\mathbf{p}, \downarrow>^+\, , \widetilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow>^- = +|\mathbf{p}, \uparrow>^+\, . \quad (51)
\end{aligned}
$$

This is due to corresponding algebraic structures of self/anti-self charge-conjugate spinors.

Next, it is possible a situation when the operators of the space inversion and charge conjugation commute each other in the Fock space [14]. For instance,

$$
\begin{aligned}
U^c_{[1/2]} U^s_{[1/2]} |\mathbf{p}, \uparrow>^+ &= +i U^c_{[1/2]} |-\mathbf{p}, \downarrow>^+ = +i|-\mathbf{p}, \downarrow>^-\, , \quad (52)\\
U^s_{[1/2]} U^c_{[1/2]} |\mathbf{p}, \uparrow>^+ &= +U^s_{[1/2]} |\mathbf{p}, \uparrow>^- = +i|-\mathbf{p}, \downarrow>^-\, . \quad (53)
\end{aligned}
$$

The second choice of the charge conjugation operator answers for the case when the $\widetilde{U}^c_{[1/2]}$ and $U^s_{[1/2]}$ operations anticommute:

$$\widetilde{U}^c_{[1/2]} U^s_{[1/2]} |\mathbf{p},\uparrow>^+ = +i\widetilde{U}^c_{[1/2]}|-\mathbf{p},\downarrow>^+ = -i|-\mathbf{p},\uparrow>^- \,, \tag{54}$$

$$U^s_{[1/2]}\widetilde{U}^c_{[1/2]}|\mathbf{p},\uparrow>^+ = -U^s_{[1/2]}|\mathbf{p},\downarrow>^- = +i|-\mathbf{p},\uparrow>^- \;. \tag{55}$$

The states $|\mathbf{p},\uparrow>^+ \pm i|\mathbf{p},\downarrow>^+$ answer for positive (negative) parity, respectively. But, what is important, *the antiparticle states* (moving backward in time) have the same properties with respect to the operation of space inversion as the corresponding *particle states* (as opposed to $j=1/2$ Dirac particles). The states which are eigenstates of the charge conjugation operator in the Fock space are $U^c_{[1/2]}\left(|\mathbf{p},\uparrow>^+ \pm i|\mathbf{p},\uparrow>^-\right) = \mp i\left(|\mathbf{p},\uparrow>^+ \pm i|\mathbf{p},\uparrow>^-\right)$. There is no any simultaneous sets of states which would be "eigenstates" of the operator of the space inversion and of the charge conjugation $U^c_{[1/2]}$.

Finally, the time reversal *anti-unitary* operator in the Fock space should be defined in such a way that the formalism to be compatible with the *CPT* theorem. If we wish the Dirac states to transform as $V(T)|\mathbf{p},\pm 1/2>=\pm|-\mathbf{p},\mp 1/2>$ we have to choose (within a phase factor), ref. [13]: $S(T)=\begin{pmatrix}\Theta_{[1/2]} & 0\\ 0 & \Theta_{[1/2]}\end{pmatrix}$. Thus, in the first relevant case we obtain for the $\Psi(x^\mu)$ field, Eq. (35):

$$V^{^T} a^\dagger_\uparrow(\mathbf{p})(V^{^T})^{-1} = a^\dagger_\downarrow(-\mathbf{p})\,, V^{^T} a^\dagger_\downarrow(\mathbf{p})(V^{^T})^{-1} = -a^\dagger_\uparrow(-\mathbf{p})\,, \tag{56}$$

$$V^{^T} b_\uparrow(\mathbf{p})(V^{^T})^{-1} = b_\downarrow(-\mathbf{p})\,, V^{^T} b_\downarrow(\mathbf{p})(V^{^T})^{-1} = -b_\uparrow(-\mathbf{p})\,. \tag{57}$$

The analogs of the above equations in the $(1,0)\oplus(0,1)$ representation space are:

$$C_{[1]} = e^{i\theta_c}\begin{pmatrix}0 & \Theta_{[1]}\\ -\Theta_{[1]} & 0\end{pmatrix}\,,\quad \Theta_{[1]} = \begin{pmatrix}0 & 0 & 1\\ 0 & -1 & 0\\ 1 & 0 & 0\end{pmatrix}\,, \tag{58}$$

$$P = e^{i\theta_s}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}R = e^{i\theta_s}\gamma_{00}R\,, \Gamma^5 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\,. \tag{59}$$

One can define the $\Gamma^5 C$ self/anti-self charge conjugate 6-component objects.

$$\Gamma^5 C_{[1]}\lambda(p^\mu) = \pm\lambda(p^\mu)\,, \Gamma^5 C_{[1]}\rho(p^\mu) = \pm\rho(p^\mu)\,. \tag{60}$$

The $C_{[1]}$ matrix is constructed from dynamical equations for the charged spin-1 particles. They are also NOT the eigenstates of the parity operator (except for $\lambda_\rightarrow$):

$$P\lambda^S_\uparrow = +\lambda^S_\downarrow\,, P\lambda^S_\rightarrow = -\lambda^S_\rightarrow\,, P\lambda^S_\downarrow = +\lambda^S_\uparrow\,, \tag{61}$$

$$P\lambda^A_\uparrow = -\lambda^A_\downarrow\,, P\lambda^A_\rightarrow = +\lambda^A_\rightarrow\,, P\lambda^A_\downarrow = +\lambda^A_\uparrow\,. \tag{62}$$

The dynamical equations are

$$\gamma_{\mu\nu}p^\mu p^\nu \lambda^S_{\uparrow\downarrow} - m^2\lambda^S_{\downarrow\uparrow} = 0\,, \gamma_{\mu\nu}p^\mu p^\nu\lambda^A_{\uparrow\downarrow} + m^2\lambda^A_{\downarrow\uparrow} = 0\,, \tag{63}$$

$$\gamma_{\mu\nu}p^\mu p^\nu\lambda^S_\rightarrow + m^2\lambda^S_\rightarrow = 0\,, \gamma_{\mu\nu}p^\mu p^\nu\lambda^A_\rightarrow - m^2\lambda^A_\rightarrow = 0\,. \tag{64}$$

Under the appropriate choice of the basis and phase factors we have $\rho^S_{\uparrow\downarrow} = +\lambda^S_{\downarrow\uparrow}$, $\rho^A_{\uparrow\downarrow} = -\lambda^A_{\downarrow\uparrow}$, $\rho^S_{\rightarrow} = -\lambda^S_{\rightarrow}$, $\rho^A_{\rightarrow} = +\lambda^S_{\rightarrow}$. On the secondary quantization level we obtained similar results as in the spin-1/2 case.

The conclusions are: 1) The momentum-space Majorana -like spinors are considered in the $(j,0)\oplus(0,j)$ representation space. 2) They have different properties from the Dirac spinors even on the classical level. 3) It is convenient to work in the 8-dimensional space. Then, we can impose the Gelfand-Tsetlin-Sokolik (Bargmann-Wightman-Wigner) prescription of 2-dimensional representation of the inversion group. 4) Gauge transformations are different. The axial charge is possible. 5) Experimental differencies have been recently discussed (the possibility of observation of the phase factor/eigenvalue of the C-parity), see [8]. 6) (Anti)commutation relations are assumed to be different from the Dirac case (and the $2(2j+1)$ case) due to the bi-orthonormality of the states (the spinors are self-orthogonal). 7) The $(1,0)\oplus(0,1)$ case has also been considered. The $\Gamma^5 C$-self/anti-self conjugate objects have been introduced. The results are similar to the $(1/2,0)\oplus(0,1/2)$ representation. The 12-dimensional formalism was introduced. 8) The field operator can describe both charged and neutral states.

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REFERENCES

1. E. Majorana, *N*uovo Cimento **1**4, 171 (1937).
2. S. M. Bilenky and B. M. Pontekorvo, *P*hys. Repts **4**2, 224 (1978).
3. A. Barut and G. Ziino, *M*od. Phys. Lett. A**8**, 1099 (1993); G. Ziino, *I*nt. J. Mod. Phys. A**1**1, 2081 (1996).
4. D. V. Ahluwalia, *I*nt. J. Mod. Phys. A**1**1, 1855 (1996).
5. P. Lounesto, *C*lifford Algebras and Spinors, Cambridge University Press, Cambridge, 2002, Ch. 11 and 12; R. da Rocha and W. Rodrigues, Jr., *W*here are Elko Spinor Fields in Lounesto Spinor Field Classification? Preprint math-ph/0506075.
6. V. V. Dvoeglazov, *I*nt. J. Theor. Phys. **3**4, 2467 (1995); *N*uovo Cim. **1**08A, 1467 (1995); *H*adronic J. **2**0, 435 (1997); *A*cta Phys. Polon. B**2**9, 619 (1998).
7. V. V. Dvoeglazov, *M*od. Phys. Lett. A**1**2, 2741 (1997).
8. M. Kirchbach, C. Compean and L. Noriega, *B*eta Decay with Momentum-Space Majorana Spinors. *E*ur. Phys. J. A**2**2, 149 (2004).
9. V. V. Dvoeglazov, *I*nt. J. Theor. Phys., **4**3, 1287-1299 (2004).
10. V. V. Dvoeglazov, *I*nt. J. Theor. Phys., **3**7, 1909-1914 (1998).
11. M. Markov, ZhETF **7** 579 (1937); ibid. 603; *N*ucl. Phys. **5**5, 130 (1964).
12. I. M. Gelfand and M. L. Tsetlin, *Z*hETF **3**1, 1107 (1956); G. A. Sokolik, *Z*hETF **3**3, 1515 (1957).
13. C. Itzykson and J.-B. Zuber, *Q*uantum Field Theory, McGraw-Hill Book Co., 1980, p. 156.
14. B. Nigam and L. L. Foldy, *P*hys. Rev. **1**02, 1410 (1956).