# Metastability of protoneutron stars in the extended Zimanyi-Moszkowski model

(revised version 2.0)\*

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#### Abstract

We have investigated the protoneutron star (PNS) using the extended Zimanyi-Moszkowski relativistic mean-field model of hadronic matter with and without antikaon condensation. The trapped neutrinos in PNS delay the appearance of negative-charged hadrons  $\Xi^-$  or  $K^-$ , and so its equation of state becomes rather stiffer than neutrino-free neutron star (NS) at high baryon densities. As the results, the maximum gravitational and baryonic masses of PNS are larger than those of NS. Thus, there exists metastable PNS that might collapse to a low-mass black hole after deleptonization. However the careful analyses of hadron composition reveal that the central baryon density of the lightest metastable PNS is lower than the density above which the negative-charged hadrons appear. Consequently, the PNSs, in which the negative charges are carried by leptons only, can be also metastable.

#### 1 Introduction

The supernova explosion SN1987A [1] offered a unique opportunity to verify for the first time our knowledge of astronomy and astrophysics using various detection techniques. It was confirmed that the neutrino-trapped compact object, the protoneutron star (PNS), was initially produced and then cooled off through the radiation of neutrinos. However there is no evidence until now [1] that the deleptonized cold neutron star (NS) is formed after neutrino burst. It was therefore suggested [2] that the delayed collapse of PNS to a low-mass black hole (BH) had happened.

Such a possibility was also investigated by Bombaci [3] in terms of baryonic masses of PNS and NS because the baryon number is conserved in PNS during its cooling through deleptonization. It has been shown that the delayed collapse to BH cannot occur if the PNS and NS are composed of nucleons and leptons only. On the contrary, a low-mass BH will be formed if the negative charges in PNS and NS are carried by hadrons

<sup>\*</sup>This paper is the revised version of mp\_arc 05-243, whose programing code of numerical calculation unfortunately had a few bugs so that the spin degeneracy of neutrino was 2 rather than 1. Although the figures in section 3 are modified, our main results are not affected.

besides leptons. Because the negative charged hadrons as hyperons and antikaons have strangeness, we can say in other words that a BH will be formed if PNS and NS are composed of strange particles. In fact the collapse to a BH due to the hyperonization in PNS has been reproduced [4] by numerical simulation.

Although Bombaci used nonlinear Walecka (NLW) model [5,6] of baryon matter, his result was also reproduced by the nonrelativistic Brueckner-Hartree-Fock (BHF) calculation [7] and quark-meson coupling (QMC) model [8]. This suggests that the result of Ref. [3] does not depend on the models of baryon matter and so is universal. However the above three models cannot be regarded to be physically reasonable. The several parameters in the NLW model are fitted to reproduce the properties of saturated nuclear matter and finite nuclei. Moreover, it has no explicit or implicit density-dependences. Therefore, the NLW model is really valuable below saturation density, but we cannot believe that it is useful at higher densities.

The BHF theory is an effort to describe nuclear matter from fundamental baryon-baryon interactions. Because the relativistic BHF calculation including hyperons is not feasible at present, the nonrelativistic BHF calculation can be a standard for the density-dependent models of baryon matter. However the refined investigations in Refs. [9] and [10] predicted the maximum masses of NSs being smaller than the canonical value  $1.44M_{\odot}$  [11]. On the other hand, the QMC model is an attempt to take into account the effect of baryon structure on the properties of medium. It is based on a bag model of baryons and has implicit density dependence. The improved version of it works relatively well [12,13] for dense baryon matter. However there is strong suspicion against its basic concept that the inside of a bag is fulfilled by the scalar mean-field of baryon matter.

Consequently, there is a room for studying other models of dense baryon matter. In this work we reexamine the results of Refs. [3], [7] and [8] in terms of a new nonlinear relativistic mean-field model developed in Refs. [14] and [15]. Our model is an extension of the Zimanyi-Moszkowski (ZM) model [16] based on the constituent quark picture of baryons. It takes into account the effect of baryon structure in the medium as the QMC model does. The effect is reflected in the effective renormalized meson-baryon coupling constants that depend on the scalar mean-fields. In other words, the baryon-baryon interactions in the medium are determined self-consistently. This is physically reasonable because the baryons in the mean-field theory of dense medium are quasi-particles. In the relativistic mean-field (RMF) theory, the quasi-particles are characterized by their reduced masses from the free values owing to the scalar potentials. Therefore, the interactions between the quasi-particles should depend on their effective masses.

#### 2 The extended ZM model

Our model Lagrangian of PNS matter composed of baryon octets and leptons is the same as Ref. [14]. We can also add the contribution of condensed antikaons [15]. The present

work has neglected the effect of finite temperature in the interior of PNS. This is because it is not crucial for a possibility of the delayed collapse in Ref. [3] and because our result is compared with the calculations of Refs. [7] and [8] that are performed at T = 0 MeV. The resultant energy density becomes

$$\mathcal{E} = \frac{1}{4} \sum_{\substack{B=p,n,\Lambda,\Sigma^{+},\\ \Sigma^{0},\Sigma^{-},\Xi^{0},\Xi^{-}}} (3E_{BF}^{*}\rho_{B} + M_{B}^{*}\rho_{BS}) + \frac{1}{4} \sum_{l=e,\mu^{-},\nu_{e},\nu_{\mu}} (3E_{lF}\rho_{l} + m_{l}\rho_{lS}) + \sum_{\bar{K}=\bar{K}^{0},K^{-}} M_{\bar{K}}^{*}\rho_{\bar{K}} + \frac{1}{2} m_{\sigma}^{2} \langle \sigma \rangle^{2} + \frac{1}{2} m_{\omega}^{2} \langle \omega_{0} \rangle^{2} + \frac{1}{2} m_{\delta}^{2} \langle \delta_{3} \rangle^{2} + \frac{1}{2} m_{\rho}^{2} \langle \rho_{03} \rangle^{2} + \frac{1}{2} m_{\sigma^{*}}^{2} \langle \sigma^{*} \rangle^{2} + \frac{1}{2} m_{\phi}^{2} \langle \phi_{0} \rangle^{2},$$

$$(1)$$

where  $\rho_B$  and  $\rho_l$  are the vector densities of the baryons and leptons in PNS matter. The  $\rho_{BS}$  and  $\rho_{lS}$  are their scalar densities.  $E^*_{BF}$  and  $E_{lF}$  are their Fermi energies. The  $m_l$  is the mass of each lepton. (Of course we assume  $m_{\nu_e} = m_{\nu_\mu} = 0$ .) The  $\rho_{\bar{K}}$  is the density of condensed antikaon. The  $m_\sigma$  etc. are masses of the mesons and  $\langle \sigma \rangle$  etc. are their mean-fields.

The effective masses  $M_B^*$  and  $M_{\bar{K}}^*$  of each baryon and antikaon are

$$M_B^* = M_B - g_{BB\sigma}^* \langle \sigma \rangle - g_{BB\delta}^* \langle \delta_3 \rangle I_{3B} - g_{BB\sigma^*}^* \langle \sigma^* \rangle, \qquad (2)$$

$$M_{\bar{K}}^* = M_K - g_{\bar{K}\bar{K}\sigma}^* \langle \sigma \rangle - g_{\bar{K}\bar{K}\delta}^* I_{3\bar{K}} \langle \delta_3 \rangle - g_{\bar{K}\bar{K}\sigma^*}^* \langle \sigma^* \rangle, \tag{3}$$

where  $I_{3B} = \{1, -1, 0, 1, 0, -1, 1, -1\}$  for  $B = \{p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-\}$  and  $I_{3\bar{K}} = \{1, -1\}$  for  $\bar{K} = \{\bar{K}^0, K^-\}$ .  $M_B$  is free baryon mass and  $M_K = 495.7 \,\text{MeV}$  is free kaon mass. The renormalized scalar-meson coupling constants  $g^*_{BB(\bar{K}\bar{K})\sigma}$  etc. are found in Refs. [14] and [15]. On the other hand, the vector potential  $V_B$  and  $V_{\bar{K}}$  are given by

$$V_B = g_{BB\omega}^* \langle \omega_0 \rangle + g_{BB\rho}^* \langle \rho_{03} \rangle I_{3B} + g_{BB\phi}^* \langle \phi_0 \rangle, \qquad (4)$$

$$V_{\bar{K}} = -g_{\bar{K}\bar{K}\omega}^* \langle \omega_0 \rangle + g_{\bar{K}\bar{K}o}^* \langle \rho_{03} \rangle I_{3\bar{K}} - g_{\bar{K}\bar{K}\phi}^* \langle \phi_0 \rangle, \qquad (5)$$

where  $g_{BB(\bar{K}\bar{K})\omega}^*$  etc. are the renormalized vector-meson coupling constants [14,15]. The vector mean-fields are given by

$$\langle \omega_0 \rangle = \sum_B \frac{g_{BB\omega}^*}{m_\omega^2} \, \rho_B - \sum_{\bar{K}} \frac{g_{\bar{K}\bar{K}\omega}^*}{m_\omega^2} \, \rho_{\bar{K}}, \tag{6}$$

$$\langle \rho_{03} \rangle = \sum_{B} I_{3B} \frac{g_{BB\rho}^*}{m_{\rho}^2} \rho_B + \sum_{\bar{K}} I_{3\bar{K}} \frac{g_{\bar{K}\bar{K}\rho}^*}{m_{\rho}^2} \rho_{\bar{K}},$$
 (7)

$$\langle \phi_0 \rangle = \sum_Y \frac{g_{YY\phi}^*}{m_\phi^2} \rho_Y - \sum_{\bar{K}} \frac{g_{\bar{K}\bar{K}\phi}^*}{m_\phi^2} \rho_{\bar{K}}.$$
 (8)

The scalar mean-fields and so the effective masses and coupling constants of all the

baryons are expressed by the three effective masses of proton, neutron and lambda [14,15]. Their values are determined by minimizing the energy density by them. The resultant three equations [14] are solved under the chemical equilibrium condition, total baryon and lepton number conservations and charge neutral condition [17]:

$$\mu_i = b_i \,\mu_n - q_i \left(\mu_e - \mu_{\nu_e}\right),\tag{9}$$

$$\mu_{\mu} - \mu_{\nu_{\mu}} = \mu_{e} - \mu_{\nu_{e}},\tag{10}$$

$$\rho_T = \sum_{\substack{B=p, n, \Lambda, \Sigma^+, \\ \Sigma^0, \Sigma^-, \Xi^0, \Xi^+}} \rho_B, \tag{11}$$

$$Y_{Le} = \frac{\rho_e + \rho_{\nu_e}}{\rho_T}, \quad Y_{L\mu} = \frac{\rho_\mu + \rho_{\nu_\mu}}{\rho_T},$$
 (12)

$$\sum_{i=B,\bar{K},l} q_i \, \rho_i = 0, \tag{13}$$

where  $\mu_i$  is the chemical potential of each baryon and antikaon,  $b_i$  and  $q_i$  are the corresponding baryon number and charge, and  $\rho_T$  is the total baryon density. Usually,  $Y_{Le} \simeq 0.4$  and  $Y_{L\mu} = 0$  [17] are assumed.

# 3 Numerical analyses

First, we investigate PNS without antikaon condensation because it is unlikely to exist in cold neutrino-free NSs [15,18]. We employ the same meson-baryon coupling constants as the set 4 in Table 1 of Ref. [14]. They are consistent to the most recent information on hyperon interactions from the experimental and theoretical investigations of hypernuclei, and so can be regarded as the most reliable choices at present. In this respect it is noted that the meson baryon coupling constants in Refs. [3] and [8] are not physically reasonable.

Figure 1 shows the particle fractions  $\rho_i/\rho_T$  as functions of  $\rho_T$ . The neutrino fraction initially decreases as  $\rho_T$  increases. It however turns to increase as  $\Lambda$  appears above  $\rho_T = 0.452 \, \mathrm{fm^{-3}}$ . Appearances of  $\Xi^-$  above  $\rho_T = 0.584 \, \mathrm{fm^{-3}}$  and  $\Xi^0$  above  $\rho_T = 0.680 \, \mathrm{fm^{-3}}$  further increase neutrino fraction. This can be readily understood from the chemical equilibrium condition  $\mu_p = \mu_n - \mu_e + \mu_{\nu_e}$  because the abundance of hyperons decrease neutron fraction owing to baryon number conservation. Especially, the  $\Xi^-$  also increases proton fraction and decreases electron fraction from charge neutral condition. In this respect we note that in the BHF calculation of Ref [7] the abundance of  $\Xi$ s are replaced by  $\Sigma$ s. This however suggests that the baryon-baryon interactions employed in Ref. [7] are not physically reasonable because it has been recently confirmed [19] that the potential of  $\Sigma$  in nuclear matter is repulsive.

We have stopped the calculation of Fig. 1 at  $\rho_T = 1.046 \, \mathrm{fm^{-3}}$  above which there are

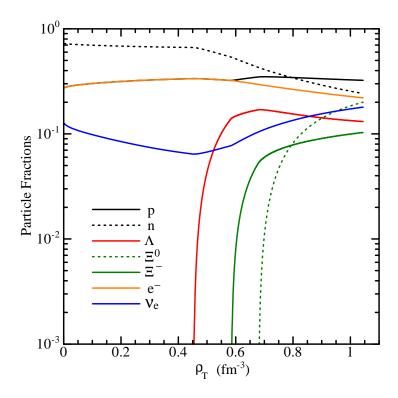


Figure 1: The particle fractions  $\rho_i/\rho_T$  in PNS without antikaon condensation as functions of the total baryon density  $\rho_T$ .

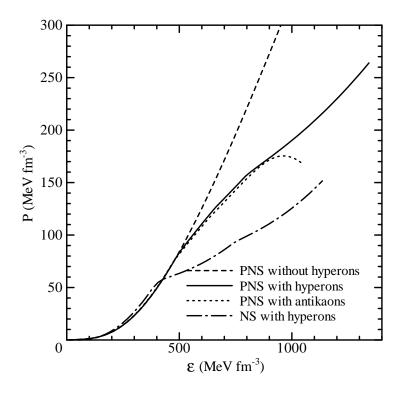


Figure 2: The equations of state for neutrino-trapped PNS containing hyperons but no antikaons (solid), PNS containing antikaons as well as hyperons (dotted), PNS composed of only nucleons and leptons (dashed) and neutrino-free NS containing hyperons but no antikaons (dotted-dashes).

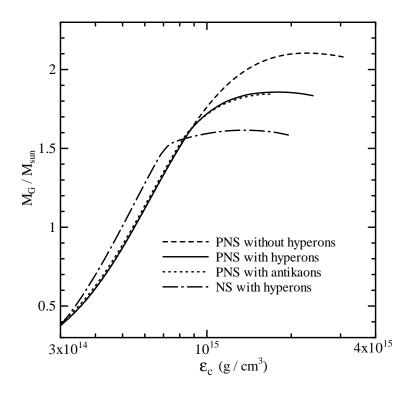


Figure 3: The gravitational masses (in units of solar mass) of neutrino-trapped PNS and neutrino-free NS. The curves are the same as Fig. 2.

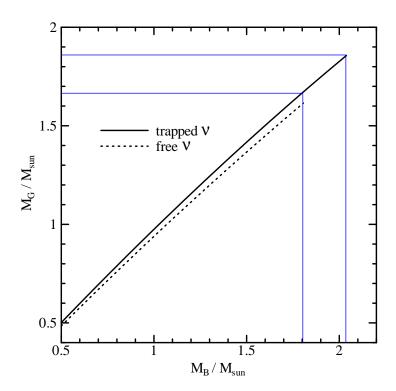


Figure 4: The gravitational  $(M_G)$ -baryonic  $(M_B)$  mass correlation (in units of solar mass) of neutrino-trapped PNS (solid) and neutrino-free NS (dotted) with hyperons but no antikaons. The blue lines indicate the maximum and minimum baryonic masses and the corresponding gravitational masses of metastable PNS.

no chemical equilibrium states. This is never unphysical because as mentioned above our baryon-baryon interactions in medium are determined self-consistently. In other words, the chemical equilibrium, charge neutral and lepton number conservation conditions restrict the baryon-baryon interactions in dense medium. Moreover, it is expected that the RMF model of baryons partially loses its physical meaning above  $\rho_T \simeq 1.0 \, \mathrm{fm}^{-3}$ . In this respect it is noted that Ref. [8] considers quark matter core in PNS and NS. However there remain significant uncertainties in the theory of deconfined quark phase.

The equation of state (EOS) and the gravitational mass sequence obtained by integrating the Tolman-Oppenheimer-Volkov (TOV) equation [20] are shown by the solid curves in Figs. 2 and 3. We have employed the EOSs in low-density region  $\rho_T \leq 0.08 \, \mathrm{fm^{-3}}$  by Feynman-Metropolis-Teller, Baym-Pethick-Sutherland and Negele-Vautherin from Ref. [21]. The results for the PNS composed of only nucleons and leptons and the neutrino-free NS containing hyperons are also shown by the dashed and dotted-dashed curves. The maximum gravitational masses of PNS without hyperons, PNS with hyperons and NS with hyperons are  $2.104 M_{\odot}$ ,  $1.857 M_{\odot}$  and  $1.615 M_{\odot}$ , respectively. We can see that the hyperonization and deleptonization of PNS soften the EOS. Although the stiffer EOS of PNS than NS is due in part to the pressure by trapped neutrinos, its main reason is the delayed appearance of  $\Xi^-$  in PNS. In fact, as compared with the composition of neutrino-free NS in Fig. 7 of Ref. [14], the neutral hyperons  $\Lambda$  and  $\Xi^0$  in PNS appear at almost the same densities as those in NS while the appearance of  $\Xi^-$  in PNS is delayed by abundance of neutrinos because of the chemical equilibrium condition  $\mu_{\Xi^-} = \mu_n + \mu_e - \mu_{\nu_e}$ .

Next, according to Ref. [3], we show in Fig. 4 the gravitational-baryonic mass correlation of PNS (solid) and NS (dotted) containing hyperons. It is seen that the PNSs with baryonic masses between  $1.81M_{\odot}$  and  $2.04M_{\odot}$  can be stabilized by trapped-neutrino effect and so might collapse to a BH after deleptonization. For a comparison Fig. 5 shows the same relation but for PNS and NS composed of only nucleons and leptons. In this case the maximum mass of PNS is lighter than NS and so no metastability occurs in PNS. From these results it seems reasonable to derive the same conclusion as Bombaci [3] that the delayed collapse to a low-mass BH will be possible only if the negative charges in PNS are carried by hadrons, the  $\Xi^-$  in this case, besides leptons. However, before jumping to the conclusion, we have to analyze our results carefully. For the purpose, the solid curve in Fig. 6 shows the baryonic mass of the PNS with hyperons as a function of its central baryon density. It is found that the central density  $\rho_T = 0.483 \, \mathrm{fm}^{-3}$  of the lightest metastable PNS, corresponding to the outer blue line in Fig. 4, is lower than the density above which the  $\Xi^-$  appears in Fig. 1. (The central density of the PNS with the maximal baryonic mass  $2.038M_{\odot}$  is  $\rho_T = 0.818 \, \mathrm{fm}^{-3}$ .) Therefore, the PNSs without  $\Xi^-$ , in which the negative charges are carried by leptons only and the strangeness is carried by  $\Lambda$  only, will cool down to cold NS or will collapse to low-mass BH. The conclusion in Ref. [3] is not always satisfied.

Finally, for completeness the PNS matter containing antikaon condensation is inves-

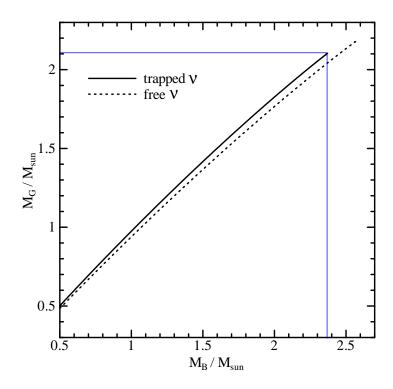


Figure 5: The same as Fig. 4 but for PNS and NS composed of only nucleons and leptons. The blue line indicates the maximum baryonic and corresponding gravitational mass of PNS.

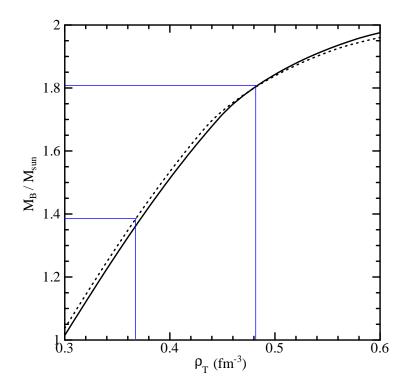


Figure 6: The baryonic masses of PNSs with and without antikaons as functions of their central total baryon density. The curves are the same as Fig. 2. The outer and inner blue lines correspond to the lightest metastable PNSs in Figs. 4 and 8, respectively.

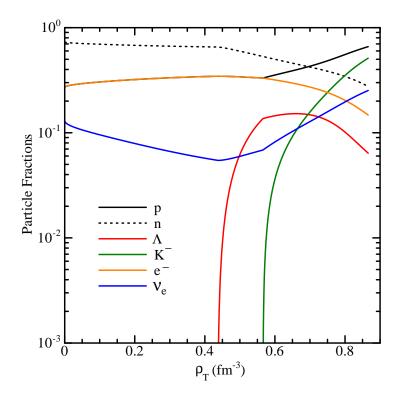


Figure 7: The same as Fig. 1 but for PNS with antikaon condensation as well as hyperons.

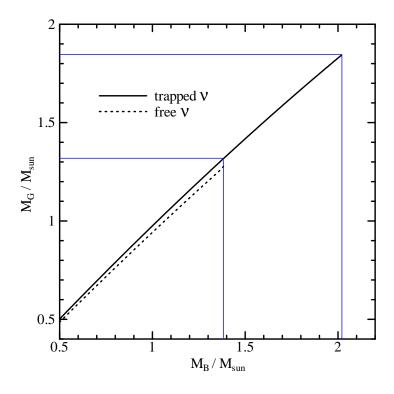


Figure 8: The same as Fig. 4 but for PNS and NS with antikaon condensation as well as hyperons. The blue lines indicate the maximum and minimum baryonic masses and the corresponding gravitational masses of metastable PNS.

tigated further. We have employed the same coupling constants as the calculation of the solid curves in figures of Ref. [15]. Figure 7 shows the composition of the PNS as function of the total baryon density. We can see the similar features to Fig. 1 except that  $K^-$  replaces  $\Xi$ s. In the comparison with Fig. 1 of Ref. [15], it is found that the appearance of  $K^-$  in PNS is also delayed by trapped neutrinos because of the chemical equilibrium condition  $\mu_{K^-} = \mu_e - \mu_{\nu_e}$ . The  $\bar{K}^0$  does not appear both in PNS and NS. Since there is no contribution of  $K^-$  to the pressure of hadronic medium, the abundance of  $K^-$  decreases the pressure above  $\rho_T = 0.80 \, \mathrm{fm^{-3}}$  as seen from the EOS of the dotted curve in Fig. 2. In the calculation of the dotted curve in Fig. 3 we have cut off the EOS at the density because there are no chemical equilibrium states above  $\rho_T = 0.86 \, \mathrm{fm^{-3}}$ . Thus, the maximum gravitational mass of PNS appears at the end point of the curve, that is  $1.844 M_{\odot}$ . The difference between the solid and dotted curves in Fig. 3 is small. This indicates that  $\Xi$ s and  $K^-$  play the same role in PNS. We can therefore expect the metastability of PNS containing antikaon condensation as was seen in Fig. 4.

Figure 8 shows the gravitational-baryonic mass correlation of PNS and NS containing antikaon condensation. It is seen that the PNSs with baryonic masses between  $1.38M_{\odot}$  and  $2.02M_{\odot}$  can be stabilized by trapped neutrino and so might collapse to a low-mass BH after deleptonization. However, as in the above case without antikaon condensation, we have to analyze the hadron composition carefully again. The dotted curve in Fig. 6 shows the baryonic mass of PNS containing  $K^-$  as a function of its central baryon density. It is found that the central density  $\rho_T = 0.368 \, \mathrm{fm}^{-3}$  of the lightest metastable PNS, corresponding to the inner blue line in Fig. 8, is lower than the density above which the  $\Lambda$  appears in Fig. 7. Therefore, the PNS composed of only nucleons and leptons may be metastable. However, the result is never consistent to Fig. 5 and so strongly suggests [15,18] that the antikaon condensed phase is unlikely to exist in NS matter.

# 4 Summary

We have reinvestigated the possibility of the delayed collapse to low-mass BH of PNS. The new fully self-consistent nonlinear relativistic mean-field model of hadronic matter developed in Refs. [14] and [15] is employed. The model is more realistic than the NR-BHF, NLW and QMC models used in the previous works. In addition, our meson-baryon coupling constants are physically more reasonable than their choices. We have confirmed that the trapped neutrinos delay the appearance of negative-charged hadrons  $\Xi^-$  or  $K^-$  in PNS, and so its EOS becomes rather stiffer than neutrino-free NS at high baryon densities. Consequently, the maximum gravitational and baryonic masses of PNS are larger than those of NS. Thus, there exists metastable PNS that might collapse to a low-mass BH after deleptonization. However the careful analyses of hadron composition reveal that the PNS without the negative-charged hadrons can be stabilized by trapped neutrinos. We have also confirmed the result of Ref. [15] that the antikaon condensed

phase is unlikely to exist in NS matter.

The formation of low-mass BH in SN1987A is still a controversial problem [22] due to the lack of observational information. There is another possibility of the delayed formation of BH by matter fallback mechanism in a supernova. Moreover, we cannot exclude the possibility that an unobserved NS not BH is formed in the supernova remnant. In the respect of the present work, we have to investigate further the evolution of hot PNS [23] so as to verify the applicability of our EOS for dense hadronic matter to SN1987A. It is the subject of a future work.

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