A new nonlinear mean-field model of neutron star matter*

K. Miyazaki

Abstract

A new relativistic mean-field model of neutron star matter is developed. It is a generalization of the Zimanyi-Moszkowski (ZM) model based on the constituent quark picture of baryons. The renormalized meson-hyperon coupling constants in medium are uniquely determined in contrast to the naive extention of ZM model and so the application of the model to high-density neutron star (NS) matter is possible. Our results of the particle composition and the mass-radius relation of NSs agree well with those obtained from the phenomenologically-determined realistic equation-of-state.

1 Introduction

The successes of the relativistic models [1] have covered a wide range of diverse phenomena of nuclear structure and scatterings. Although it is now established that the nuclear physics in low-density region is well controlled by the relativistic dynamics, this is somewhat odd because the relativistic effects are expected to become more crucial at high densities. One of the objects to be appropriate for investigating nuclear physics at high density is the high-energy heavy-ion collision. However the realistic description of the reaction dynamics involves serious problems regardless of the relativistic or nonrelativistic model.

Another object to be suitable for high-density nuclear physics is the neutron star (NS) matter. Since Glendenning [2] has first applied the relativistic mean-field (RMF) model to it, many efforts have been done until now. Recent the most refined and detailed investigations using the nonlinear (NLW) self-coupling model are the works of Refs. [3] and [4]. It is however noted that many parameters in the NLW model are adjusted to normal nuclear matter and finite nuclei. Therefore the NLW model is really valid at low densities but there are no guarantees that it is useful at high densities. In this respect, the models embodying explicit or implicit density-dependence are desired. Although the Dirac-Brueckner-Hartree-Fock (DBHF) theory [5] is such an effort to be the most elaborate, it is known that the DBHF theory spoils thermodynamic consistency [6]. This problem can be avoided by reducing the DBHF to the mean-field approximation, that

*This paper is the revised version of CDS ext-2004-010. Some texts and mistypes have been corrected.

is, the density-dependent hadron field (DDRH) theory. It has been also applied to the NS matter in Ref. [7]. However, in contrast to the nonrelativistic Brueckner-Hartree-Fock (NRBHF) theory [8,9], there are no realistic DBHF calculations of baryon matter including hyperons at present. As a result, the density dependence of the meson-hyperon vertices in the DDRH model cannot be well determined.

Therefore the other RMF models, which exhibit density dependence but are independent of the DBHF theory, are desired. Zimanyi and Moszkowski (ZM) [10] developed one of such models. It has the renormalized $NN\sigma$ coupling constant,

$$g^*_{NN\sigma} = (\, M^*_N/M_N)\, g_{NN\sigma}, \tag{1}$$

where M_N is the nucleon mass and M^*_N is its effective mass in nuclear matter. In the extended version of the ZM model, the $NN\omega$ coupling constant is also renormalized as

$$g^*_{NN\omega} = (\, M^*_N/M_N)\, g_{NN\omega}. \tag{2}$$

These renormalized coupling constants are density-dependent through the effective mass. It is important that they are determined self-consistently as well as the effective mass in the nuclear matter. This feature is also realized in the DBHF theory but not in the DDRH model. However the ZM model cannot reproduce the saturation properties of nuclear matter. Moreover, its extension to hyperons is obscure. Nevertheless, applications to the ZM model to NS have been attempted in Refs. [11] and [12].

Here we recall another model, which also exhibits density dependence but is independent of the DBHF theory, the quark-meson coupling (QMC) model [13]. This model also has the $NN\sigma$ coupling constant to depend on the scalar mean-field. Although the original QMC model fails to reproduce nuclear matter saturation properties, its modified version works well and has been applied to NS matter in Ref. [14]. Because the QMC model is based on the bag model of nucleons, it can be extended to hyperons in an unambiguous way. It is valuable to note that the original version of the QMC model predicts the similar saturation properties of nuclear mater to the ZM model. This suggests that the ZM model has its theoretical origin in nucleon structure.

Such a speculation has first been investigated in Ref. [15], which exhibits that the relativistic SU(6) model of meson-nucleon couplings reproduces the similar $NN\sigma$ coupling to the ZM model. If the structure of a nucleon is effective on nuclear matter properties, we expect that the contribution of the so-called Z-graph, which is an essential ingredient of the relativistic models, should be suppressed. This expectation has also been investigated in Ref. [16]. We have found another model that has the effective renormalized coupling constants to depend on both the scalar and vector mean-fields and reproduces the similar saturation properties of nuclear matter to the DBHF calculation. Furthermore Ref. [17] has investigated the effect of the meson cloud of nucleons in nuclear medium. It has been found that the wave-function renormalization of nuclear nucleons leads to the effective

$NN\sigma$ and $NN\omega$ coupling constants, which become the generalization of the ZM model and reproduce the similar results to the DBHF calculation.

Unfortunately, it is not easy to extend the above works to isospin asymmetric matter including hyperons. Then the author reanalyzed [18] the ZM model from a picture of the constituent quark model (CQM) and found a modified ZM model to reproduce the similar result to Ref. [17]. Because the model is based on the CQM, it can be extended to include isovector mean-fields and hyperons in an unambiguous way in contrast to the naive extention of ZM model. Therefore it has been readily applied to strange hadronic matter in Ref. [18] and then to charge asymmetric nuclear matter in Ref. [19].

The purpose of the present work is to extend the model in Refs. [18] and [19] to NS matter including all baryon octets. In the next section, we first introduce the effective renormalized meson-baryon coupling constants, and then employ them in the RMF approximation. In section 3 the model is applied to non-rotating cold β-stable NSs and the numerical results are discussed. Finally we summarize our investigation and draw conclusions in section 4.

2 Formalism

In this work, we consider the contributions of the isoscalar scalar meson σ, isoscalar vector meson ω, isovector vector meson ρ and isovector scalar meson $\delta\,[a_0(980)]$. The (hidden) strange meson considered in Ref. [3] is not taken into account because the interactions between hyperons are not well known. The masses of baryons are assumed to be $M_N = 938.9\,\mathrm{MeV}$, $M_\Lambda = 1115.6\,\mathrm{MeV}$, $M_\Sigma = 1193.05\,\mathrm{MeV}$ and $M_\Xi = 1318.1\,\mathrm{MeV}$.

2.1 Renormalized coupling constants.

The renormalized meson(Π)-nucleon(N) coupling constants $g^*_{NN\Pi}$ have already been derived in Ref. [19]. They are related to the free coupling constant $g_{NN\Pi}$ as

$$g^*_{pp\sigma(\omega)} = \left[(1-\lambda_N) + \lambda_N m^*_p\right] g_{NN\sigma(\omega)}, \tag{3}$$

$$g^*_{nn\sigma(\omega)} = \left[(1-\lambda_N) + \lambda_N m^*_n\right] g_{NN\sigma(\omega)}, \tag{4}$$

$$g^*_{pp\delta(\rho)} = \left[(1-\lambda_N) + \lambda_N\left(2m^*_n - m^*_p\right)\right] g_{NN\delta(\rho)}, \tag{5}$$

$$g^*_{nn\delta(\rho)} = \left[(1-\lambda_N) + \lambda_N\left(2m^*_p - m^*_n\right)\right] g_{NN\delta(\rho)}, \tag{6}$$

where the renormalization constant λ_N is

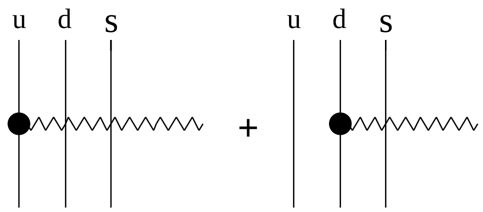
$$\lambda_N = 1/3. \tag{7}$$

The quantities $m^*_{p\,(n)}$ is the ratio of the effective nucleon mass $M^*_{p(n)}$ in medium to the free mass:

$$m^*_{p\,(n)} = M^*_{p\,(n)}/M_N = \left(M_N + S_{p(n)}\right)/M_N, \tag{8}$$

where $S_{p\,(n)}$ is the scalar potential of proton (neutron). M^*_p and M^*_n are different from each other owing to the isovector scalar mean-field by $\delta\,[a_0(980)]$ meson. Because the renormalized coupling constants depend on the effective masses, they are determined self-consistently in nuclear medium.

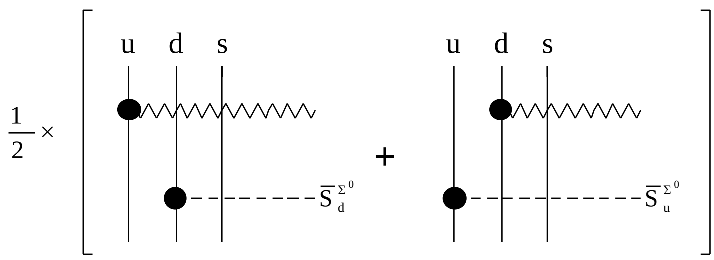
Although we have applied the above model to the RMF calculation of neutron star in Ref. [19], the effects of hyperons have not been considered. It is known [20] that the hyperons play crucial role in high-density region in NSs. Therefore we have to extend our model to hyperons. The special case of the isospin symmetric strange hadronic matter has been investigated in Ref. [18]. The extension to isospin asymmetric matter is straightforward. For the first example, the Σ^0 hyperon is considered. In the QCM, the free $\Sigma^0\Sigma^0\sigma$ or $\Sigma^0\Sigma^0\omega$ coupling is schematically written by



or expressed by

$$g_{\Sigma^0\Sigma^0\sigma(\omega)} = g_{\Sigma\Sigma\sigma(\omega)} = 2\,g^{\Sigma}_{qq\sigma(\omega)}. \tag{9}$$

The wavy lines denote σ or ω mesons and q is u or d quark. Although one of the u and d quarks is the spectator in the above diagram, both the quarks in the Σ^0 hyperon embedded in baryon matter feel the mean-field. In the RMF model of baryon matter, the mass of Σ^0 hyperon in the medium must be reduced by the scalar mean-field as the mass of nucleons in nuclear matter. In the QCM this means that both the masses of u and d quarks are also reduced by the scalar potential. Therefore we can consider the following medium correction to $g_{\Sigma^0\Sigma^0\sigma(\omega)}$:



The dashed lines are the effects of the mean-fields on quarks defined by

$$\bar{S}^{\Sigma^0}_{u(d)} = S^{\Sigma^0}_{u(d)}/M_{\Sigma^0}, \tag{10}$$

where $S_u^{\Sigma^0}$ and $S_d^{\Sigma^0}$ are the scalar potentials of u and d quarks in the Σ^0 hyperon.

Adding the above correction to Eq. (9), we have the renormalized (or effective) $\Sigma^0\Sigma^0\sigma(\omega)$ coupling constant $g^*_{\Sigma^0\Sigma^0\sigma(\omega)}$,

$$g^*_{\Sigma^0\Sigma^0\sigma(\omega)} = g_{\Sigma\Sigma\sigma(\omega)} + \frac{1}{2}\left(\bar{S}_u^{\Sigma^0} + \bar{S}_d^{\Sigma^0}\right) g^{\Sigma}_{qq\sigma(\omega)} = \left[1 + \frac{1}{4}\left(\bar{S}_u^{\Sigma^0} + \bar{S}_d^{\Sigma^0}\right)\right] g_{\Sigma\Sigma\sigma(\omega)}. \tag{11}$$

Because the scalar potential of Σ^0 hyperon is given by

$$S_{\Sigma^0} = S_u^{\Sigma^0} + S_d^{\Sigma^0}, \tag{12}$$

Eq. (11) is rewritten as

$$g^*_{\Sigma^0\Sigma^0\sigma(\omega)} = \left[(1-\lambda_\Sigma) + \lambda_\Sigma m^*_{\Sigma^0}\right] g_{\Sigma\Sigma\sigma(\omega)}, \tag{13}$$

$$\lambda_\Sigma = 1/4, \tag{14}$$

where the effective mass of the Σ^0 hyperon $M^*_{\Sigma^0} = m^*_{\Sigma^0} M_\Sigma$ is introduced by using

$$\bar{S}_{\Sigma^0} = S_{\Sigma^0}/M_\Sigma = \bar{S}_u^{\Sigma^0} + \bar{S}_d^{\Sigma^0} = m^*_{\Sigma^0} - 1. \tag{15}$$

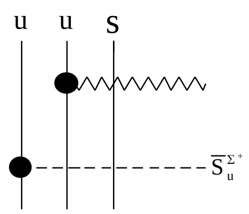
Of course, the renormalized $\Lambda\Lambda\sigma(\omega)$ coupling constant $g^*_{\Lambda\Lambda\sigma(\omega)}$ is given by the similar expression:

$$g^*_{\Lambda\Lambda\sigma(\omega)} = \left[(1-\lambda_\Lambda) + \lambda_\Lambda m^*_\Lambda\right] g_{\Lambda\Lambda\sigma(\omega)}, \tag{16}$$

$$\lambda_\Lambda = 1/4. \tag{17}$$

The effective mass m^*_Y of Λ and other hyperons is defined by the same way as $m^*_{\Sigma^0}$ and will be determined in section 2.2.

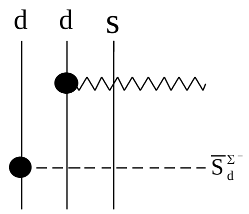
Next, the charged Σ's are considered. The medium correction to $\Sigma^+\Sigma^+\Pi$ ($\Pi = \sigma$, ω, δ and ρ) coupling constant is given by the following graph:



According to the same procedure as Σ^0, the renormalized coupling constant $g^*_{\Sigma^+\Sigma^+\Pi}$ is readily obtained.

$$g^*_{\Sigma^+\Sigma^+\Pi} = \left[(1-\lambda_\Sigma) + \lambda_\Sigma m^*_{\Sigma^+}\right] g_{\Sigma\Sigma\Pi}. \tag{18}$$

Similarly, the medium correction to $\Sigma^-\Sigma^-\Pi$ coupling constant is depicted by



The renormalized coupling constants $g^*_{\Sigma^-\Sigma^-\Pi}$ is given by

$$g^*_{\Sigma^-\Sigma^-\Pi} = [(1-\lambda_\Sigma) + \lambda_\Sigma m^*_{\Sigma^-}]\, g_{\Sigma\Sigma\Pi}. \tag{19}$$

The effective masses of Σ^+, Σ^0 and Σ^- are different from each other because of the isovector scalar mean-field. Consequently, the renormalized coupling constants of the Σ hyperons break the isospin symmetry.

Since the Ξ hyperons contain only one u or d quark, there are no medium corrections to the $\Xi\Xi\Pi$ couplings in our model. This indicates that our model is not fully consistent. We have to take into account the contributions of the (hidden) strange mesons considered in Ref. [3] to construct a fully consistent model in which the $\Xi\Xi\Pi$ couplings are also renormalized. Unfortunately, there is little reliable information about the interactions between hyperons. Therefore we defer the investigation of the effect by strange mesons for future works. For later convenience, the renormalized $\Xi\Xi\Pi$ coupling constants are formally introduced in the same forms as the other hyperons:

$$g^*_{\Xi^0\Xi^0\Pi} = [(1-\lambda_\Xi) + \lambda_\Xi\, m^*_{\Xi^0}]\, g_{\Xi\Xi\Pi}. \tag{20}$$

$$g^*_{\Xi^-\Xi^-\Pi} = [(1-\lambda_\Xi) + \lambda_\Xi\, m^*_{\Xi^-}]\, g_{\Xi\Xi\Pi}. \tag{21}$$

$$\lambda_\Xi = 0, \tag{22}$$

In our model, the strangeness (S) of the baryons explicitly appears as the different value of the renormalization constant λ_B, that is $\lambda_B = 1/3$ for $S = 0$, $\lambda_B = 1/4$ for $S = -1$ and $\lambda_B = 0$ for $S = -2$. The naïve extension of the ZM model to hyperons in Ref. [12] corresponds to using $\lambda_Y = 1$ for all the hyperons.

2.2 The RMF model of NS matter

Using the renormalized meson-baryon coupling constants obtained above, our mean-field model Lagrangian for NS matter becomes

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{\substack{B=p,n,\Lambda,\Sigma^+,\\ \Sigma^0,\Sigma^-,\Xi^0,\Xi^-}} \bar{\psi}_B \left(\not{p} - M^*_B - \gamma^0 V_B\right)\psi_B + \sum_{l=e^-,\mu^-} \bar{\psi}_l \left(\not{p} - m_l\right)\psi_l \\ & - \frac{1}{2}\, m_\sigma^2 \left\langle \sigma \right\rangle^2 + \frac{1}{2}\, m_\omega^2 \left\langle \omega_0 \right\rangle^2 - \frac{1}{2}\, m_\delta^2 \left\langle \delta_3 \right\rangle^2 + \frac{1}{2}\, m_\rho^2 \left\langle \rho_{03} \right\rangle^2 ,\end{aligned} \tag{23}$$

where ψ_B and ψ_l are the Dirac fields of the baryons and the leptons, $\langle\sigma\rangle$, $\langle\omega_0\rangle$, $\langle\delta_3\rangle$ and $\langle\rho_{03}\rangle$ are the mean-fields, m_l is the mass of each lepton, and m_σ, m_ω, m_δ and m_ρ are the masses of each meson. The effective mass M_B^* of each baryon is

$$M_B^* = M_B + S_B. \tag{24}$$

The scalar potentials S_B is given by

$$S_B = -g_{BB\sigma}^* \langle\sigma\rangle - g_{BB\delta}^* \langle\delta_3\rangle I_{3B}, \tag{25}$$

where $I_{3B} = \{1, -1, 0, 1, 0, -1, 1, -1\}$ for $B = \{p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-\}$. The vector potential V_B of each baryon is given by

$$V_B = g_{BB\omega}^* \langle\omega_0\rangle + g_{BB\rho}^* \langle\rho_{03}\rangle I_{3B}. \tag{26}$$

From Eq. (25), the scalar mean-fields are expressed by the effective masses of the nucleons.

$$\langle\sigma\rangle = -\frac{M_N}{g_{NN\sigma}} \frac{A^{(0)}}{C^{(0)}}, \tag{27}$$

$$\langle\delta_3\rangle = -\frac{M_N}{g_{NN\delta}} \frac{B^{(0)}}{C^{(0)}}, \tag{28}$$

where

$$A^{(0)} = h_{nn\delta}^* \left(m_p^* - 1\right) + h_{pp\delta}^* \left(m_n^* - 1\right), \tag{29}$$

$$B^{(0)} = h_{nn\sigma}^* \left(m_p^* - 1\right) - h_{pp\sigma}^* \left(m_n^* - 1\right), \tag{30}$$

$$C^{(0)} = h_{pp\sigma}^* h_{nn\delta}^* + h_{nn\sigma}^* h_{pp\delta}^*, \tag{31}$$

and we have abbreviated Eqs. (3)-(6) as

$$g_{pp\,(nn)\Pi}^* = h_{pp\,(nn)\Pi}^* \, g_{NN\Pi} \quad (\Pi = \sigma,\, \omega,\, \delta \text{ and } \rho)\,, \tag{32}$$

Therefore the effective masses of the hyperons (Y) can be expressed by the effective masses of the nucleons (N). After some manipulations, we obtain

$$1 - m_Y^* = \left[\lambda_Y - \frac{M_Y}{M_N} \frac{C^{(0)}}{r_{Y\sigma} A^{(0)} + r_{Y\delta} B^{(0)} I_{3Y}}\right]^{-1}, \tag{33}$$

where $m_Y^* = M_Y^*/M_Y$, $r_{Y\sigma(\delta)} = g_{YY\sigma(\delta)}/g_{NN\sigma(\delta)}$. The vector potentials of hyperons are also expressed by V_p and V_n, and so the vector potentials of every baryon are unified in a single expression:

$$V_B = \frac{g^*_{BB\omega} g^*_{nn\rho} + I_{3B} g^*_{BB\rho} g^*_{nn\omega}}{g^*_{pp\omega} g^*_{nn\rho} + g^*_{nn\omega} g^*_{pp\rho}} V_p + \frac{g^*_{BB\omega} g^*_{pp\rho} - I_{3B} g^*_{BB\rho} g^*_{pp\,\omega}}{g^*_{pp\omega} g^*_{nn\rho} + g^*_{nn\omega} g^*_{pp\rho}} V_n. \tag{34}$$

Then the energy density of NS matter is written by

$$\begin{aligned}
\mathcal{E} \;=\; & \sum_{\substack{B=p,n,\Lambda,\Sigma^+,\\ \Sigma^0,\Sigma^-,\Xi^0,\Xi^-}} \left(\langle E^*_k \rangle_B + V_B \right) \rho_B + \sum_{l=e^-,\mu^-} \langle E_k \rangle_l \, \rho_l \\
& + \frac{1}{2} M_N^2 \left(\frac{m_\sigma}{g_{NN\sigma}} \right)^2 \left(\frac{A^{(0)}}{C^{(0)}} \right)^2 + \frac{1}{2} M_N^2 \left(\frac{m_\delta}{g_{NN\delta}} \right)^2 \left(\frac{B^{(0)}}{C^{(0)}} \right)^2 \\
& - \frac{1}{2} m_\omega^2 \left(\frac{g^*_{nn\rho} V_p + g^*_{pp\rho} V_n}{g^*_{pp\,\omega} g^*_{nn\rho} + g^*_{nn\omega} g^*_{pp\rho}} \right)^2 - \frac{1}{2} m_\rho^2 \left(\frac{g^*_{nn\omega} V_p - g^*_{pp\,\omega} V_n}{g^*_{pp\,\omega} g^*_{nn\rho} + g^*_{nn\omega} g^*_{pp\rho}} \right)^2 ,
\end{aligned} \tag{35}$$

where $\langle E^*_k \rangle_B$ and $\langle E_k \rangle_l$ are the average kinetic energies of the baryons and the leptons, and ρ_B and ρ_l are their vector densities. The vector potentials V_p and V_n are determined by $\partial \mathcal{E} / \partial V_p = 0$ and $\partial \mathcal{E} / \partial V_n = 0$. Consequently, we have a general expression of the vector potentials:

$$V_B = \sum_{B'} \left(\frac{g^*_{BB\omega} g^*_{B'B'\omega}}{m_\omega^2} + I_{3B} I_{3B'} \frac{g^*_{BB\rho} g^*_{B'B'\rho}}{m_\rho^2} \right) \rho_{B'} . \tag{36}$$

Substituting Eq. (36) into Eq. (35), the energy density becomes

$$\begin{aligned}
\mathcal{E} \;=\; & \left(\sum_B f_B \langle E^*_k \rangle_B \right) \rho_T + \sum_l \langle E_k \rangle_l \, \rho_l + \frac{1}{2} M_N^2 \left(\frac{m_\sigma}{g_{NN\sigma}} \right)^2 \left(\frac{A^{(0)}}{C^{(0)}} \right)^2 \\
& + \frac{1}{2} M_N^2 \left(\frac{m_\delta}{g_{NN\delta}} \right)^2 \left(\frac{B^{(0)}}{C^{(0)}} \right)^2 + \frac{1}{2} \left(\sum_B f_B \frac{g^*_{BB\omega}}{m_\omega} \right)^2 \rho_T^2 + \frac{1}{2} \left(\sum_B I_{3B} f_B \frac{g^*_{BB\rho}}{m_\rho} \right)^2 \rho_T^2 ,
\end{aligned} \tag{37}$$

where ρ_T is the total baryon density and f_B is the fraction of each baryon. The effective masses m^*_p and m^*_n are determined by solving the self-consistency equations $\partial \mathcal{E} / \partial m^*_p = 0$ and $\partial \mathcal{E} / \partial m^*_n = 0$ simultaneously. From Eq. (37) they are

$$\begin{aligned}
& \sum_B \frac{\partial m^*_B}{\partial m^*_p} \frac{M_B}{M_N} \frac{\rho_{BS}}{\rho_T} + \left(\frac{m_\sigma}{g_{NN\sigma}} \right)^2 \frac{M_N}{\rho_T} \frac{A^{(0)} \left(A^{(1)}_p C^{(0)} - A^{(0)} C^{(1)}_p \right)}{\left(C^{(0)} \right)^3} \\
& + \left(\frac{m_\delta}{g_{NN\delta}} \right)^2 \frac{M_N}{\rho_T} \frac{B^{(0)} \left(C^{(0)} - B^{(0)} C^{(1)}_p \right)}{\left(C^{(0)} \right)^3} + \left(\sum_B \lambda_B f_B \frac{g_{BB\omega}}{m_\omega} \frac{\partial m^*_B}{\partial m^*_p} \right) \left(\sum_B f_B \frac{g^*_{BB\omega}}{m_\omega} \right) \frac{\rho_T}{M_N} \\
& + \left(\sum_B I_{3B} \lambda_B f_B \frac{g_{BB\rho}}{m_\rho} \frac{\partial \tilde{m}^*_B}{\partial m^*_p} \right) \left(\sum_B I_{3B} f_B \frac{g^*_{BB\rho}}{m_\rho} \right) \frac{\rho_T}{M_N} = 0,
\end{aligned} \tag{38}$$

$$
\sum_B \frac{\partial m_B^*}{\partial m_n^*} \frac{M_B}{M_N} \frac{\rho_{BS}}{\rho_T} + \left(\frac{m_\sigma}{g_{NN\sigma}}\right)^2 \frac{M_N}{\rho_T} \frac{A^{(0)}\left(A_n^{(1)}C^{(0)} - A^{(0)}C_n^{(1)}\right)}{\left(C^{(0)}\right)^3}
$$

$$
- \left(\frac{m_\delta}{g_{NN\delta}}\right)^2 \frac{M_N}{\rho_T} \frac{B^{(0)}\left(C^{(0)} + B^{(0)}C_n^{(1)}\right)}{\left(C^{(0)}\right)^3} + \left(\sum_B \lambda_B f_B \frac{g_{BB\omega}}{m_\omega} \frac{\partial m_B^*}{\partial m_n^*}\right) \left(\sum_B f_B \frac{g_{BB\omega}^*}{m_\omega}\right) \frac{\rho_T}{M_N}
$$

$$
+ \left(\sum_B I_{3B}\lambda_B f_B \frac{g_{BB\rho}}{m_\rho} \frac{\partial \tilde{m}_B^*}{\partial m_n^*}\right) \left(\sum_B I_{3B} f_B \frac{g_{BB\rho}^*}{m_\rho}\right) \frac{\rho_T}{M_N} = 0, \tag{39}
$$

where ρ_{BS} is the scalar density of each baryon. The quantities $A_{p\,(n)}^{(1)}$ and $C_{p(n)}^{(1)}$ are defined by

$$
A_p^{(1)} = (1-\xi_N) + \xi_N\left(2m_p^* - m_n^*\right), \tag{40}
$$

$$
C_p^{(1)} = \xi_N h_{nn\delta}^*, \tag{41}
$$

$$
A_n^{(1)} = (1-\xi_N) + \xi_N\left(2m_n^* - m_p^*\right), \tag{42}
$$

$$
C_n^{(1)} = \xi_N h_{pp\delta}^*, \tag{43}
$$

with $\xi_N \equiv 2\lambda_N$. The effective masses $\tilde{m}_B^*$ are defined by

$$
\tilde{m}_p^* = 2m_n^* - m_p^*, \tag{44}
$$

$$
\tilde{m}_n^* = 2m_p^* - m_n^*, \tag{45}
$$

$$
\tilde{m}_Y^* = m_Y^*. \tag{46}
$$

(See Eqs. (5) and (6).) Therefore $\partial\tilde{m}_p^*/\partial m_p^* = -1$ *etc.* The derivatives of the effective masses of hyperons by the effective masses of nucleons are

$$
\frac{\partial m_Y^*}{\partial m_p^*} = (1-m_Y^*)^2 \frac{M_Y}{M_N} \frac{r_{Y\sigma}\left(A_p^{(1)}C^{(0)} - A^{(0)}C_p^{(1)}\right) + r_{Y\delta} I_{3Y}\left(C^{(0)} - B^{(0)}C_p^{(1)}\right)}{\left(r_{Y\sigma}A^{(0)} + r_{Y\delta}B^{(0)}I_{3Y}\right)^2}, \tag{47}
$$

$$
\frac{\partial m_Y^*}{\partial m_n^*} = (1-m_Y^*)^2 \frac{M_Y}{M_N} \frac{r_{Y\sigma}\left(A_n^{(1)}C^{(0)} - A^{(0)}C_n^{(1)}\right) - r_{Y\delta} I_{3Y}\left(C^{(0)} + B^{(0)}C_n^{(1)}\right)}{\left(r_{Y\sigma}A^{(0)} + r_{Y\delta}B^{(0)}I_{3Y}\right)^2}. \tag{48}
$$

Finally, the pressure P is given by

$$
P = \frac{1}{4}\sum_B \left(E_{BF}^*\rho_B - M_B^*\rho_{BS}\right) + \frac{1}{4}\sum_l \left(E_{lF}\rho_l - m_l\,\rho_{lS}\right) - \frac{1}{2}M_N^2\left(\frac{m_\sigma}{g_{NN\sigma}}\right)^2\left(\frac{A^{(0)}}{C^{(0)}}\right)^2
$$

$$
- \frac{1}{2}M_N^2\left(\frac{m_\delta}{g_{NN\delta}}\right)^2\left(\frac{B^{(0)}}{C^{(0)}}\right)^2 + \frac{1}{2}\left(\sum_B f_B \frac{g_{BB\omega}^*}{m_\omega}\right)^2 \rho_T^2 + \frac{1}{2}\left(\sum_B I_{3B} f_B \frac{g_{BB\rho}^*}{m_\rho}\right)^2 \rho_T^2, \tag{49}
$$

where E^*_{BF} and E_{lF} are the Fermi energy of the hyperons and the leptons in NS matter and ρ_{lS} is the scalar density of the leptons.

3 Numerical analyses

For numerical calculations of the NS matter, we first determine the free meson-baryon coupling constants. The $NN\sigma$ and $NN\omega$ coupling constants are fixed to reproduce the nuclear matter saturation [18]. We assume the saturation energy of $-15.75\,\mathrm{MeV}$ at the saturation density $0.16\,\mathrm{fm}^{-3}$. The values $(g_{NN\sigma}/m_\sigma)^2 = 16.9\,\mathrm{fm}^2$ and $(g_{NN\omega}/m_\omega)^2 = 12.5\,\mathrm{fm}^2$ are obtained. The effective nucleon mass and the incompressibility of saturated nuclear matter are $m^*_N = 0.605$ and $K = 302\,\mathrm{MeV}$ respectively. The $YY\omega$ coupling constants are fixed from the CQM or the SU(6) symmetry:

$$\frac{1}{3}\,g_{NN\omega} = \frac{1}{2}\,g_{\Lambda\Lambda\omega} = \frac{1}{2}\,g_{\Sigma\Sigma\omega} = g_{\Xi\Xi\omega}. \tag{50}$$

The $YY\sigma$ coupling constants are determined to give the hyperon potentials in saturated nuclear matter $U^{(N)}_Y$ that are compatible with experimental results for hypernuclei [3,7]:[1]

$$U^{(N)}_\Lambda = U^{(N)}_\Sigma = -30\,\mathrm{MeV} \quad \text{and} \quad U^{(N)}_\Xi = -28\,\mathrm{MeV}. \tag{51}$$

In our model, they are given by

$$U^{(N)}_Y = -g^*_{YY\sigma}\,\langle\sigma\rangle_{NM} + g^*_{YY\omega}\,\langle\omega_0\rangle_{NM}\,, \tag{52}$$

where $\langle\sigma\rangle_{NM}$ and $\langle\omega_0\rangle_{NM}$ are the mean-fields in saturated nuclear matter.

For $NN\delta$ and $NN\rho$ coupling constants, we employ the values of the Bonn A potential in Ref. [5], $(g_{NN\delta}/m_\delta)^2 = 0.39\,\mathrm{fm}^2$ and $(g_{NN\rho}/m_\rho)^2 = 0.82\,\mathrm{fm}^2$. Unfortunately, these values produce smaller symmetry energy of nuclear matter 24.6MeV than the empirical value 30 ± 4 MeV. It is well known [4] that the RMF model is not compatible to the realistic value of $NN\rho$ coupling constant and much larger values are usually employed. In this respect, the $NN\sigma$ and $NN\omega$ coupling constants also have to be compared with the Bonn A potential. Our values determined above do not largely differ from the Bonn A potential while some works use rather different values of the coupling constants. In such cases, it is commented that the coupling constants are the effective ones in nuclear medium. This statement however reveals that the model is only valid around the saturation density. On the contrary, we have introduced the renormalized coupling constants to be determined self-consistently at any density. Therefore the values of the free (unrenormalized) coupling constants should be close to their realistic ones. At present, the Bonn A potential in Ref.

[1] Although it has been recently confirmed [24] that the optical potential of Σ is repulsive, we here assume attractive one because of the comparison of our model with the other RMF models in Refs. [3,7,12,14].

[5] is only realistic model that reproduces NN interaction and nuclear matter saturation simultaneously. On the other hand, there is no reliable empirical information about nucleon-hyperon and hyperon-hyperon interactions. For the $YY\delta$ and $YY\rho$ coupling constants, we therefore chose simply,

$$g_{NN\delta} = g_{\Sigma\Sigma\delta} = g_{\Xi\Xi\delta} \quad \text{and} \quad g_{\Lambda\Lambda\delta} = 0, \tag{53}$$

$$g_{NN\rho} = g_{\Sigma\Sigma\rho} = g_{\Xi\Xi\rho} \quad \text{and} \quad g_{\Lambda\Lambda\rho} = 0. \tag{54}$$

The properties of NS matter in our model are essentially determined by m_p^* and m_n^* that are the solutions of Eqs. (38) and (39). To solve these equations, the densities of every baryon are needed at fixed total baryon density ρ_T.

$$\rho_T = \sum_{\substack{B=p,n,\Lambda,\Sigma^+,\\ \Sigma^0,\Sigma^-,\Xi^0,\Xi^+}} \rho_B. \tag{55}$$

They are determined to fulfill the β-equilibrium condition,

$$\mu_i = b_i\mu_n - q_i\mu_e, \tag{56}$$

where μ_i is the chemical potential of all the baryons and leptons (e^- and μ^-) and b_i and q_i are the corresponding baryon number and charge. There exist only two independent chemical potentials of neutron and electron, which are fixed to satisfy Eq. (55) and the charge neutral condition,

$$\sum_{i=B,l} q_i\rho_i = 0. \tag{57}$$

Once μ_n and μ_e are obtained, the other chemical potentials are also obtained from Eq. (56) and then the densities of baryons are determined from

$$\mu_B = \left(k_{BF}^2 + M_B^{*2}\right)^{1/2} + V_B. \tag{58}$$

Since the right hand side of Eq. (58) includes M_B^*, Eqs. (38), (39) and (55)~(58) have to be solved consistently.

Figure 1 shows the fraction of every baryons and leptons in cold β-stable NS matter as functions of ρ_T. At low densities, the matter consists of nucleons and leptons. The μ^- appears near the saturation density $\rho_T = 0.16\,\text{fm}^{-3}$. The hyperons appear above $\rho_T = 0.275\,\text{fm}^{-3}$. The first is the Σ^- rather than the lighter Λ because of its negative charge. As the Σ^- appears, the fractions of the leptons turn to decrease because of the charge neutral condition and the fraction of neutron decreases more rapidly. The Σ^- increases steeply until the Λ appears above $\rho_T = 0.44\,\text{fm}^{-3}$. Since the fraction of Λ also increases rapidly, the fraction of proton almost saturates. Above $\rho_T = 0.625\,\text{fm}^{-3}$ the Ξ^-

rather than the lighter Σ^0 appears because of its negative charge as in the case of Σ^- and Λ. Since the Ξ^- increases rapidly, the Σ^- turns to decrease and the fraction of μ^- becomes lower than 10^{-3}. Consequently, the fraction of proton increases slightly. The Σ^0 appears above $\rho_T = 0.73\,\mathrm{fm}^{-3}$. As the Ξ^0 appears above $\rho_T = 0.83\,\mathrm{fm}^{-3}$, the fractions of p and Λ almost saturate. The fraction of electron becomes lower than 10^{-3} above $\rho_T = 0.925\,\mathrm{fm}^{-3}$. As the Σ^+ appears finally above $\rho_T = 0.95\,\mathrm{fm}^{-3}$ owing to its positive charge, the fraction of proton turns to decrease because of the charge neutral condition. At higher densities than $\rho_T = 1.0\,\mathrm{fm}^{-3}$, the Λ exceeds the nucleons and the Ξ's exceed the Σ's. The latter however needs a proviso that our treatment of the effective meson-Ξ couplings is not consistent to the other hyperons. We have renormalized the meson-Λ and meson-Σ coupling constants but used free coupling constant ($\lambda_\Xi = 0$) for Ξ.

Figures 2 and 3 show the effective mass $m_B^* = M_B^*/M_B$ of each baryon and the renormalized coupling constant $g^*_{BB\sigma(\omega)}/g_{BB\sigma(\omega)}$ of isoscalar meson as functions of ρ_T. Because $g^*_{\Xi\Xi\sigma(\omega)}/g_{\Xi\Xi\sigma(\omega)} = 1$ in our model, it is not shown. The curves are the same as Fig. 1. As mentioned at the end of section 2.1, the renormalization constant λ_B of each baryon is fixed according to its strangeness. The differences of the strangeness between the baryons are clearly seen in Figs. 2 and 3. There are small differences between the masses of proton and neutron, between Σ^+, Σ^0 and Σ^-, and between Ξ^0 and Ξ^- owing to the isovector scalar mean-field. Because the renormalized coupling constants depend on the effective masses, the isospin symmetry of the effective interactions is also broken. In this respect we note that the baryons in the medium are not physical particles but quasi-particles.

The effective mass of neutron becomes negative above $\rho_T = 1.2\,\mathrm{fm}^{-3}$. Reference [3] has discussed the problem of negative effective mass and suggested that the hyperon-hyperon interactions mediated by the (hidden) strange mesons can resolve the problem. Indeed, the interactions between strange quarks may be important to reproduce realistic hyperon-hyperon interactions. However they are not well known at present. Furthermore it is generally believed that the RMF models of baryon matter lose their physical validity at high densities above $\rho_T = 1.0\,\mathrm{fm}^{-3}$. We have therefore stopped the calculation when negative effective masses appear.

Figure 4 shows the equation of state (EOS) for cold β-stable NS matter. The solid and dashed curves are the results with and without hyperons. The difference between them is revealed above $\mathcal{E} = 270\,\mathrm{MeV{\cdot}fm}^{-3}$ when the Σ^- appears in Fig. 1. As is well known [20], the hyperons have effect to soften the EOS considerably.

Using the EOS obtained above, we calculate non-rotating NS by integrating the Tolman-Oppenheimer-Volkov (TOV) equation [21].

$$\frac{d\,M\,(r)}{d\,r} = \frac{4\pi^2}{c^2}\,r^2\mathcal{E}\,(r)\,, \tag{59}$$

$$\frac{d\,P\,(r)}{d\,r} = -\,\frac{G}{c^2}\,\frac{\left[\,\mathcal{E}\,(r) + P\,(r)\right]\left[M\,(r) + 4\pi\,r^3 P\,(r)/c^2\right]}{r\left[\,r - 2\,GM\,(r)/c^2\right]}, \tag{60}$$

where $P\,(r)$, $\mathcal{E}\,(r)$ and $M\,(r)$ are the radial distributions of the pressure, energy and mass of the NS. Since we have employed realistic but small $NN\rho$ coupling constant, the pressure in Fig. 4 has quite small negative values below $\rho_T = 0.04\,\mathrm{fm}^{-3}$. This however is not a problem because our RMF model of baryons describes only the high-density core-region of the NS. For the outer region, we use the EOS by Feynman-Metropolis-Teller, Baym-Pethick-Sutherland and Negele-Vautherin from Ref. [22]. Figures 5 and 6 show the gravitational mass of NSs in units of the solar mass as functions of the central energy density $\mathcal{E}_C$ and the radius R. The solid and dashed curves in Fig. 5 are the results with and without hyperons. The separation of the two curves occurs when the Σ^- appears as the case of Fig. 4. The hyperons have the effect to reduce the pressure considerably. The dashed line in Fig. 6 is the upper limit of the mass of NS derived from the data of Vela pulsar [23]. Our result almost lies under it.

Finally, we compare our results, the particle composition of Fig. 1 and the mass-radius relation of Fig. 6, with other models. For this purpose we take the result obtained from the phenomenologically-determined realistic EOS in Ref. [20] as a criterion. (The EOS 2 is chosen.) The characteristic features of its particle composition are: 1) The fraction of Σ^-, which is the hyperon appearing first, decreases gradually at high densities. 2) As a result the fraction of Ξ^- exceeds Σ^-. Our model has both the features. The DDRH model [7] using phenomenological density-dependence exhibits only the first feature while the NLW model [3] and the QMC model [14] exhibit only the second. The naïve extension of the ZM model to hyperons [12] has neither. The characteristic feature of the mass-radius relation by the effective EOS is that the mass decreases slowly up to the radius $R = 13\,\mathrm{km}$ and then falls headlong to the value below $0.5M_\odot$. Although the solid curve in Fig. 6 exhibits roundness between $R = 12\,\mathrm{km}$ and $13\,\mathrm{km}$ and slight backbend between $R = 13\,\mathrm{km}$ and $14\,\mathrm{km}$, the agreement with the effective EOS is fairly good. The DDRH model has the similar feature only by using the phenomenological density dependence. The results using the NN potential models, Groningen and Bonn A, are rather different. This suggests that the density dependences from the DBHF calculations are not necessarily reliable at high densities. In this sense our model proposes another choice against the DBHF model.

4 Summary

We have introduced the renormalized effective meson-baryon coupling constants into the RMF theory of NS matter. They are derived from the analysis of the ZM model based on the CQM of baryons and have the general form $g^*_{BB\Pi} = [(1-\lambda_B) + \lambda_B m^*_B\,]g_{BB\Pi}$ where the renormalization constant λ_B is fixed according to the strangeness (S) as $\lambda_B = 1/3$ for $S = 0$, $\lambda_B = 1/4$ for $S = -1$ and $\lambda_B = 0$ for $S = -2$. In contrast to the ZM model, the renormalized meson-hyperon coupling constants are determined uniquely without introducing additional parameters. Our model has effective density-dependence but is independent to the DBHF model in contrast to the DDRH model.

We have calculated the composition and the EOS of the NS matter that is cold, β-equilibrium and neutrino-free. Then the mass-radius relation is calculated by integrating the TOV equation using the EOS. Our results enjoy agreement with the results by the effective EOS that is phenomenological but reliable. In this respect our model is fairly promising than the other models. However there remain important problems mentioned in the following.

We have emphasized the importance to use realistic meson-nucleon coupling constants in the calculation. However the CQM of baryons, on which our model is based, is never realistic. This means that the physical values of the renormalization constants λ_B may deviates from the values employed, especially for hyperons. So as to fix the problem, it is needed to merge the realistic but simple model of the structure of baryons with the RMF model of baryon matter. Unfortunately, we have no models to be appropriate for the purpose and so postpone it to future subjects.

Furthermore we have not included the interactions between the strange quarks in hyperons, which are important for the realistic reproduction of hyperon-hyperon interactions. Although the hyperon-hyperon interactions are not well known, it is worthwhile to include the effect of the (hidden) strange mesons into our model according to Ref. [3]. We want to investigate it in a future work.

References

[1] B.D. Serot and J.D. Walecka, *Advances in Nuclear Physics*, Vol. **16** (Plenum, New York, 1986).

[2] N.K. Glendenning, Astrophys. J. **293** (1985) 470.

[3] J. Schaffner and I.N. Mishustin, Phys. Rev. **C53** (1996) 1416, [arXiv:nucl-th/9506011].

[4] H. Huber, F. Weber, M.K. Weigel and Ch. Schaab, Int. J. Mod. Phys. **E7** (1998) 301, [arXiv:nucl-th/9711025].

[5] R. Brockmann and R. Machleidt, Phys. Rev. **C42** (1990) 1965.

[6] P. Czerski, A. De Pace and A. Molinari, Phys. Rev. **C65** (2002) 044317, [arXiv:nucl-th/0204008].

[7] F. Hofmann, C.M. Keil and H. Lenske, Phys. Rev. **C64** (2001) 025804, [arXiv:nucl-th/0008038].

[8] M. Baldo, G.F. Burgio and H.-J. Schulze, Phys. Rev. **C61** (2000) 055801, [arXiv:nucl-th/9912066].

[9] I. Vidaña, A. Polls, A. Ramos, L. Engvik and M. Hjorth-Jensen, Phys. Rev. **C62** (2000) 035801, [arXiv:nucl-th/0004031].

[10] J. Zimanyi and S.A. Moszkowski, Phys. Rev. **C42** (1990) 1416. It is noted that the NLW model has linear meson-baryon couplings in spite of its name. On the contrary, the ZM model is true nonlinear mean-field model.

[11] N.K. Glendenning, F. Weber and S.A. Moszkowski, Phys. Rev. **C45** (1992) 844.

[12] A.R. Taurines, C.A.Z. Vasconcellos, M. Malheiro and M. Chiapparini, Phys. Rev. **C63** (2001) 065801, [arXiv:nucl-th/0010084].

[13] K. Saito and A.W. Thomas, Phys. Lett. **B327** (1994) 9.

[14] S. Pal, M. Hanauske, I. Zakout, H. Stőker and W. Greiner, Phys. Rev. **C60** (1999) 015802, [arXiv:nucl-th/9905010].

[15] K. Miyazaki, Prog. Theor. Phys. **91** (1994) 1271.

[16] K. Miyazaki, Prog. Theor. Phys. **93** (1995) 137.

[17] K. Miyazaki, CERN Document Server (CDS) ext-2002-056 revised by Mathematical Physics Preprint Archive (mp_arc) 05-141.

[18] K. Miyazaki, CERN Document Server (CDS) ext-2003-062 revised by Mathematical Physics Preprint Archive (mp_arc) 05-178.

[19] K. Miyazaki, CERN Document Server (CDS) ext-2003-077 revised by Mathematical Physics Preprint Archive (mp_arc) 05-190.

[20] S. Balberg, I. Lichtenstadt and G.B. Cook, Astrophys. J. Suppl. **121** (1999) 515.

[21] W.D. Arnett and R.L. Bowers, Astrophys. J. Suppl. **33** (1977) 415.

[22] V. Canuto, Ann. Rev. Astr. Ap. **12** (1974) 167; **13** (1975) 335.

[23] B. Link, R.I. Epstein and J.M. Lattimer, Phys. Rev. Lett. **83** (1999) 3362.

[24] M. Kohno, Y. Fujiwara, Y. Watanabe, K. Ogata and M. Kawai, arXiv:nucl-th/0410073.

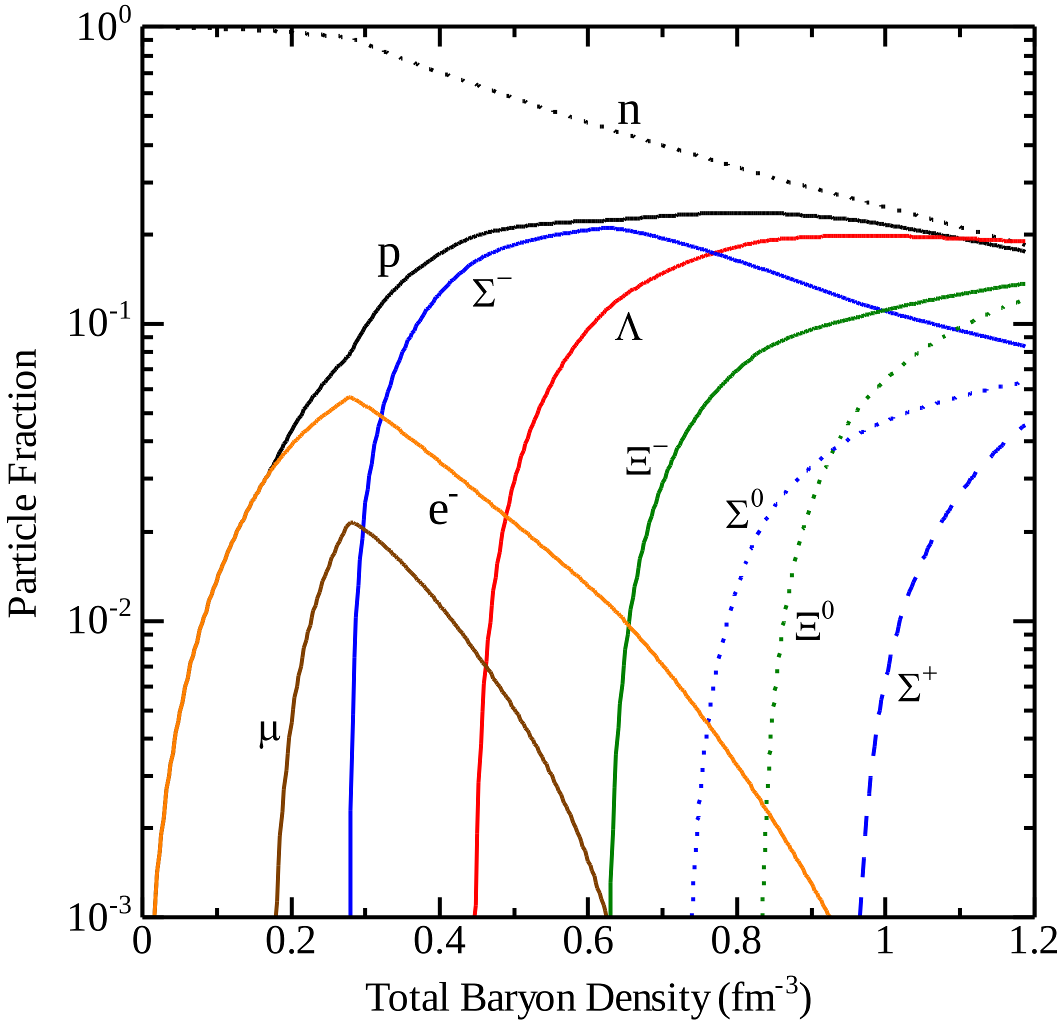


Figure 1: The partial fractions of baryons and leptons as functions of the total baryon density.

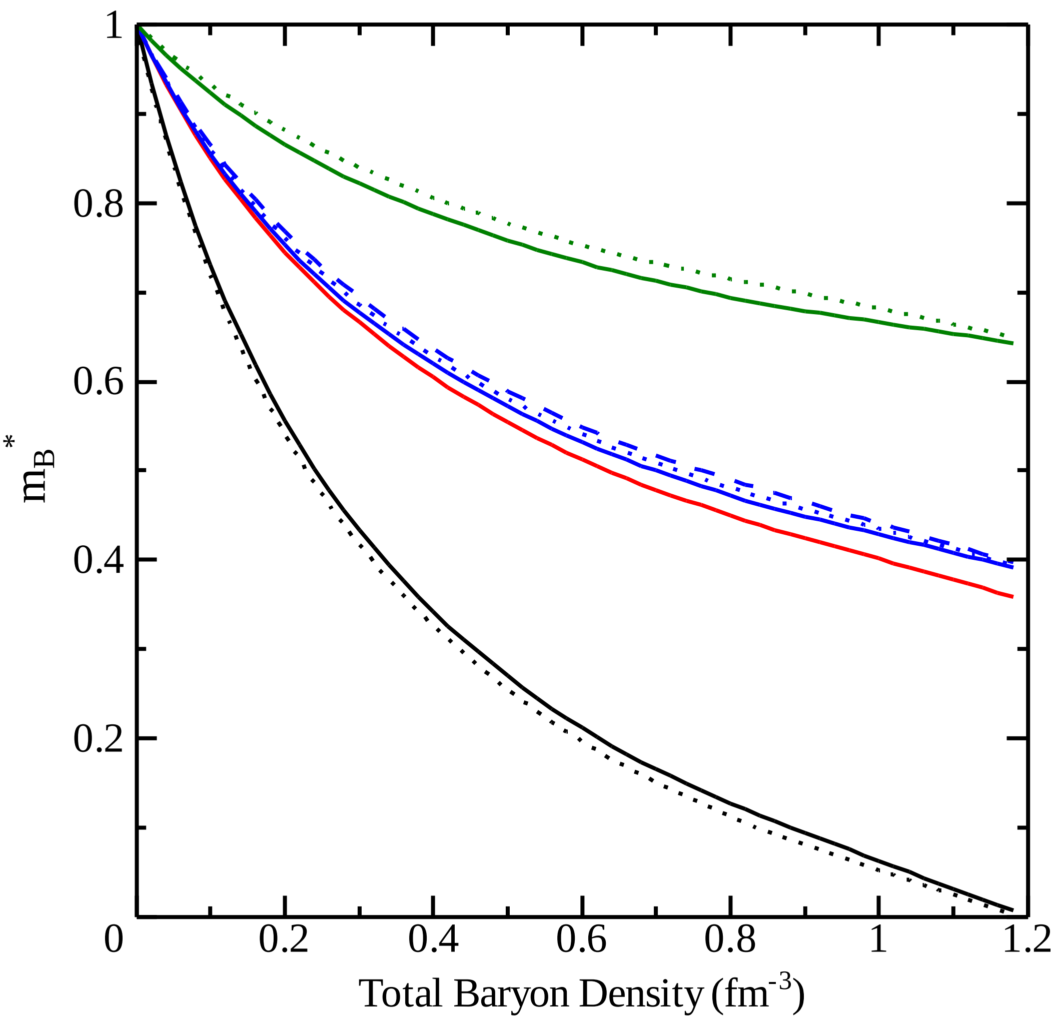


Figure 2: The effective baryon masses $m_B^* = M_B^*/M_B$ as functions of the total baryon density. The curves are the same as Fig. 1.

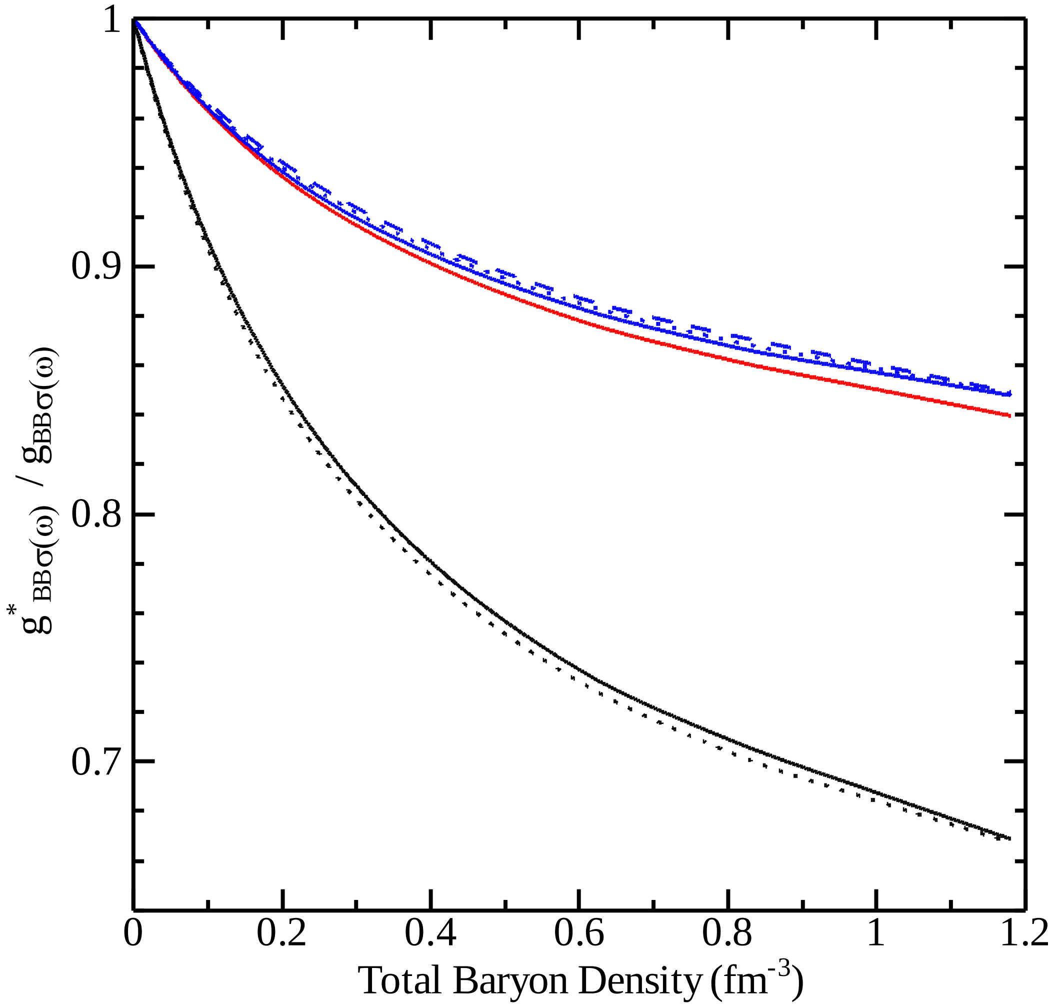


Figure 3: The renormalized coupling constants $g^*_{BB\sigma(\omega)}/g_{BB\sigma(\omega)}$ as functions of the total baryon density. The curves are the same as Fig. 1.

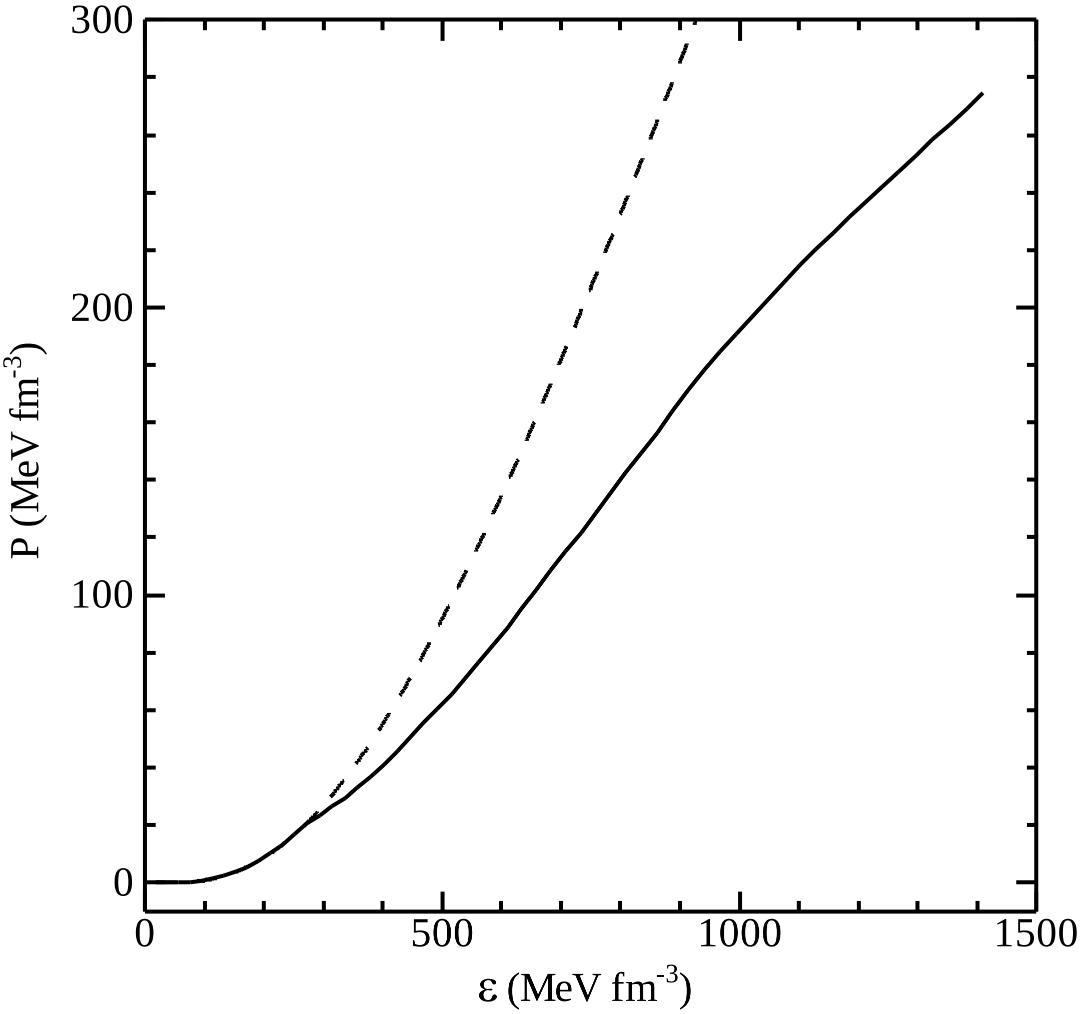


Figure 4: The equation of state for cold β-stable NS matter. The solid and dashed curves are the calculations with and without hyperons.

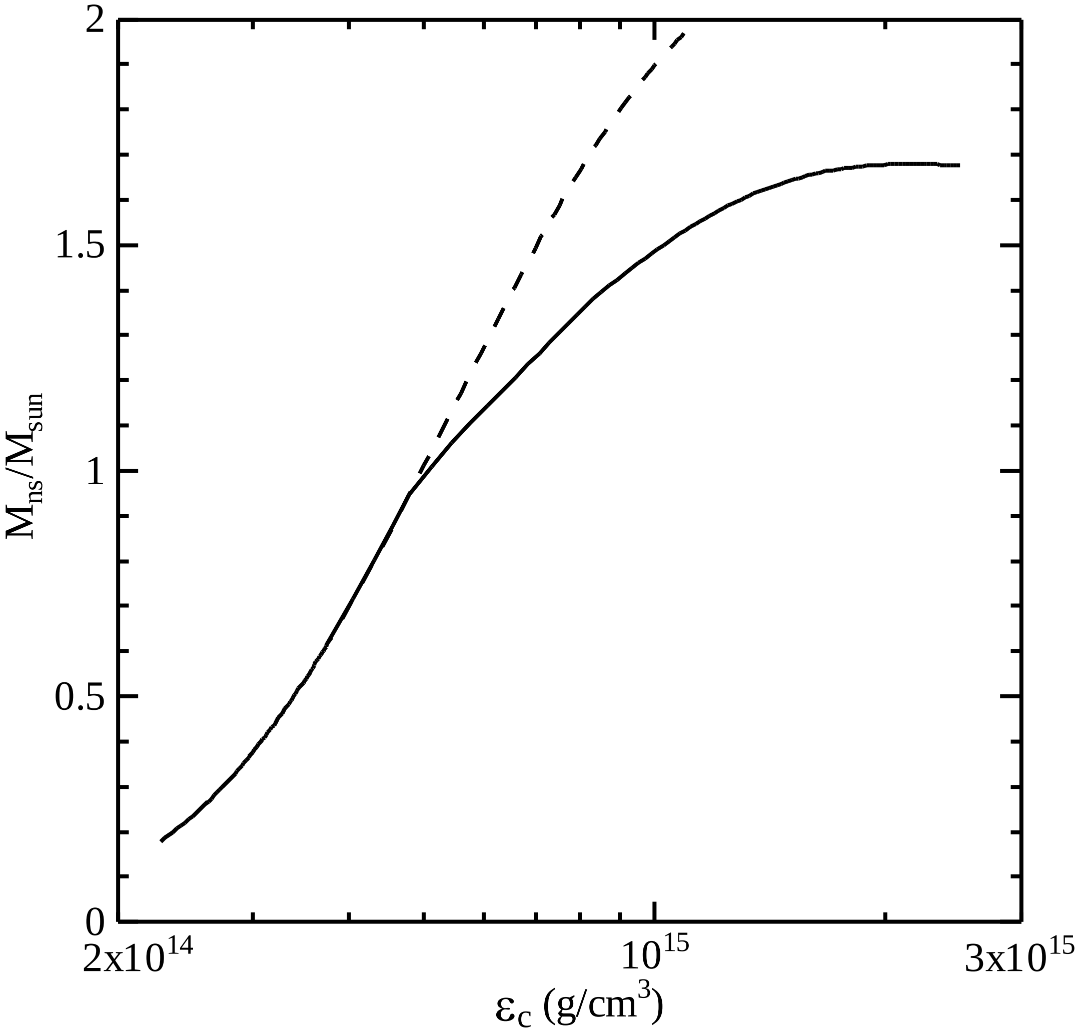


Figure 5: The NS mass in units of the solar mass as functions of the central energy density. The solid and dashed curves are the calculations with and without hyperons.

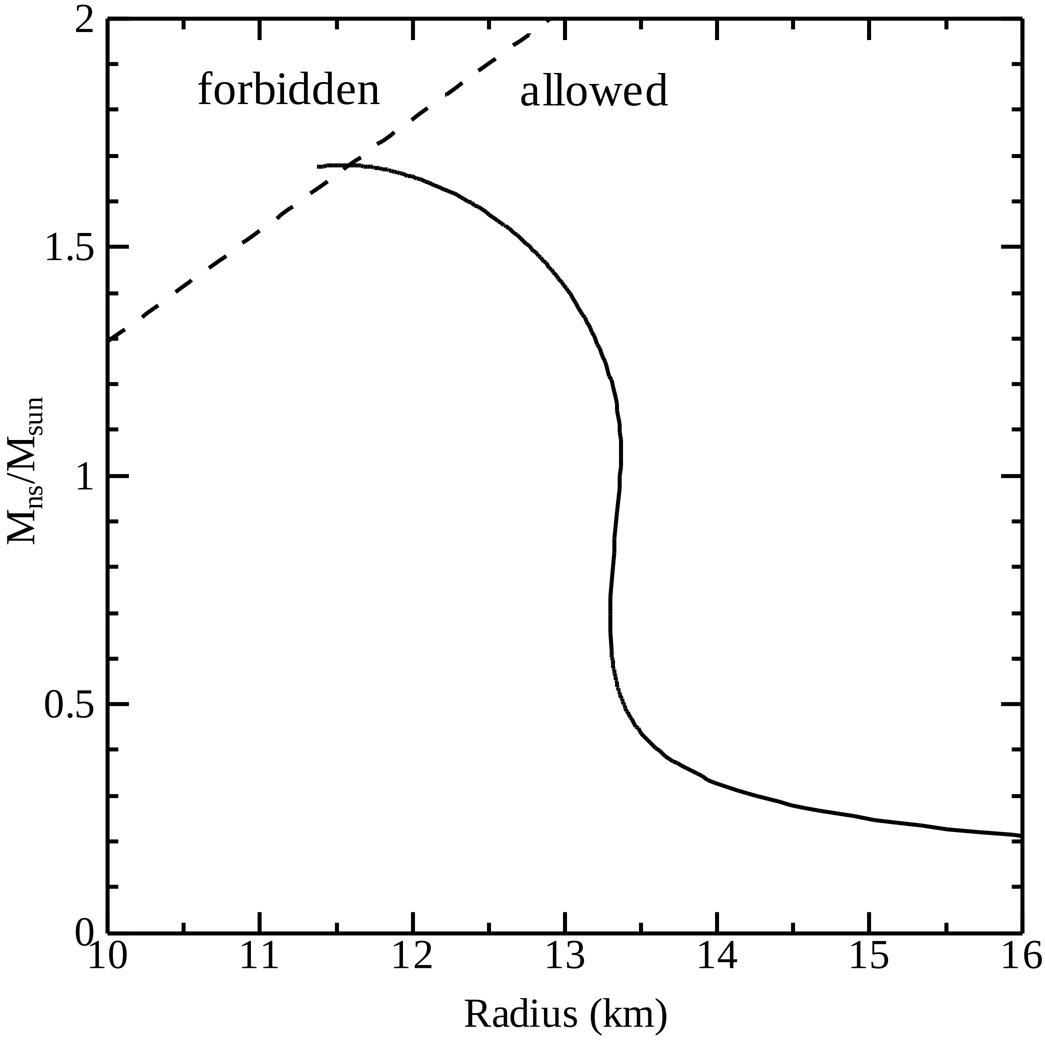


Figure 6: The NS mass in units of the solar mass versus the NS radius. The dashed line is the upper limit of the mass derived in Ref. [23].