The non-linear mean-field model of nuclear matter I
— effect of meson cloud on saturation*—

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Abstract

We investigate the effect of the meson cloud of nucleon on saturation properties of nuclear matter. Quantum correction to the scalar and vector potentials in the Walecka model is taken into account. It leads to the renormalized wave function of a nucleon in the medium, or the dressed nucleon by the meson cloud. Consequently, the $NN\sigma$ and $NN\omega$ coupling constants are renormalized. The renormalization constant can be related to the anomalous magnetic moment. The resultant renormalized Walecka model is able to reproduce nuclear matter saturation properties well.

1 Introduction

During the last twenty years, the relativistic models for nucleus have been largely developed and had great successes [1-5]. The essential ingredients of the models are the shifts of mass and energy of a nucleon in the nuclear medium by the strong negative scalar potential (or the mean σ-meson field $\langle\sigma\rangle$) and the strong positive vector potential (or the mean ω-meson field $\langle\omega\rangle$). This has been already shown by the original work by Walecka [1]. Recently Birse [6] showed more general result, a low energy theorem for a nucleon in the mean scalar and vector fields. Up to the second order of the mean fields, the effective mass M^* and the energy $E(\mathbf{p})$ of the nucleon were generally expressed by

$$M^* = M - g_{NN\sigma}\langle\sigma\rangle + \frac{1}{2}\xi_s\langle\sigma\rangle^2 + \frac{\xi_v + \xi_p M^2}{2M}\langle\omega_0\rangle^2, \tag{1}$$

$$\begin{aligned} E(\mathbf{p}) &= \varepsilon(\mathbf{p}) + \left(g_{NN\omega} + \xi_{sv}\langle\sigma\rangle\right)\langle\omega_0\rangle \\ &\quad + \frac{M}{\varepsilon(\mathbf{p})}\left[-g_{NN\sigma}\langle\sigma\rangle + \frac{1}{2}\xi_s\langle\sigma\rangle^2 + \frac{\xi_v + \xi_p\varepsilon(\mathbf{p})^2}{2M}\langle\omega_0\rangle^2\right] + \frac{|\mathbf{p}|^2}{2\,\varepsilon(\mathbf{p})^3}g_{NN\sigma}^2\langle\sigma\rangle^2, \end{aligned} \tag{2}$$

where M and $\varepsilon(\mathbf{p})$ are the mass and the energy of a free nucleon of momentum $\mathbf{p}$. These expressions, which are derived from the general dispersion relation, rely only on

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the covariance of the dynamics. They are independent of models and hold for both composite and elementary particle. Birse referred to the quantities ξ's as *polarizabilities* of the nucleon by the meson fields. They are identically zero in the Walecka model and so can be regarded as the first-order correction to it.

Here we note the model developed by Zimanyi and Moszkowski (ZM) [7]. They employed the derivative scalar coupling $\bar{\psi}\left(\not{p}/M\right)\psi\sigma$ rather than the usual one $\bar{\psi}\psi\sigma$. This model, called as the DSC model, is equivalent to employing a renormalized $NN\sigma$ coupling constant $g^*_{NN\sigma} = \left(M^*/M\right) g_{NN\sigma}$ in the Walecka model. Using this, the relation between M^* and $\langle\sigma\rangle$ is

$$m^* \equiv M^*/M = 1 - m^*\bar{\sigma} \approx 1 - \bar{\sigma} + \bar{\sigma}^2, \tag{3}$$

where $\bar{\sigma} = g_{NN\sigma}\langle\sigma\rangle/M$. The third term corresponds to *the scalar polarizability* term in Eq. (1). The DSC model can reproduce the empirical incompressibility of nuclear matter in contrast to the Walecka model.

However it is not able to reproduce the properties of finite nuclei. Koepf *et al.* [8] pointed out that this failure is due to relatively large effective mass $m^* \approx 0.85$ in the DSC model, compared to $m^* \approx 0.55$ in the Walecka model. The small effective mass or large scalar potential is necessary for large spin-orbit splitting [9]. The Dirac-Brueckner-Hartree-Fock (DBHF) calculation [10] also yields rather small effective mass $m^* \approx 0.6$. The DSC model only produces relatively weak scalar and vector potentials. One of the reasons of this shortcoming may be strong renormalization of $NN\sigma$ coupling constant. It is desirable to take a weaker renormalization. In other words we have to choose $g^*_{NN\sigma}/g_{NN\sigma} > m^*$ in the range $0 \leq m^* \leq 1$. In fact, for this purpose, the hybrid derivative scalar coupling $g^*_{NN\sigma} = \left[1 + \alpha\left(m^* - 1\right)\right] g_{NN\sigma}$ was proposed [11] where $0 \leq \alpha \leq 1$ is a parameter. (A special case of $\alpha = 1/2$ is investigated in Ref. [12].) In this case Eq. (3) becomes

$$m^* \approx 1 - \bar{\sigma} + \alpha\,\bar{\sigma}^2. \tag{4}$$

It is noted that α is just the scalar polarizability ξ_s in Eq. (1). Another reason of the failure of the DSC model is that only the $NN\sigma$ coupling constant is renormalized, but the $NN\omega$ is not so. This seems to be inconsistent. The reduction of both $NN\sigma$ and $NN\omega$ coupling constants at finite density is observed in the DBHF calculation [10]. In fact, in the appendix of Ref. [7], ZM mentioned the model using $g^*_{NN\sigma} = \left(m^*\right)^{\alpha} g_{NN\sigma}$ and $g^*_{NN\omega} = \left(m^*\right)^{\beta} g_{NN\omega}$ with $0 \leq \alpha, \beta \leq 1$. The special cases, $\alpha = 1$, $\beta = 0.5$, $\alpha = \beta = 1$ and others were studied in Ref. [13]. In this case, the effective mass, or the scalar potential, is also given by Eq. (4). On the other hand, the vector potential is expressed by

$$v \equiv V/M \approx \bar{\omega}_0 - \beta\,\bar{\sigma}\,\bar{\omega}_0, \tag{5}$$

where $\bar{\omega}_0 = g_{NN\omega}\langle\omega_0\rangle/M$. The second term is just *the mixed scalar-vector polarizability* ξ_{sv} in Eq. (2).

It has been seen that the DSC model and its extensions introduce the polarizability into the Walecka model. However, their physical origins are obscure. Here we note that the nucleon is dressed by meson cloud in the field theory of mesons and nucleon. That is, the wave function of a nucleon is renormalized. The Walecka model does not take into account this renormalization. In the present work, we investigate the effect of the meson cloud of nucleon in nuclear matter and show that the polarizabilities are due to its effect. In the next section, the quantum correction to the classical mean meson fields is studied and the dressed nucleon by meson cloud is introduced. In Section 3 the wave-function renormalization of the nucleon is related to its anomalous magnetic moment. In Section 4 the Walecka model is generalized to the renormalized nucleon and the polarizability is derived. We then calculate the properties of symmetric nuclear matter in Section 5. In the last section, we summarize the studies.

2 Dressed nucleon by meson cloud in medium

Now we consider charge symmetric nuclear matter. We suppose that the propagator of a nucleon in the medium satisfies the Dyson equation

$$G\left(p\right) = G^{(0)}\left(p\right) + G^{(0)}\left(p\right) U\, G\left(p\right). \tag{6}$$

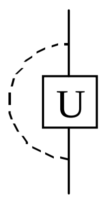
Here the potential U does not depend on the momentum p. It is composed of the scalar and vector part:

$$U = S + \gamma^0 V. \tag{7}$$

$G^{(0)}\left(p\right)$ is the nucleon propagator in the noninteracting Fermi gas and is compose of Feynman and density-dependent part [1]:

$$G^{(0)}\left(p\right) = G_F^{(0)}\left(p\right) + G_D^{(0)}\left(p\right). \tag{8}$$

Then we will investigate the quantum correction to the *classical* potential U. First, for simplicity, the following correction is studied.



$$\Gamma\left(p\right) = \sum_i \int \frac{d^4k}{\left(2\pi\right)^4} D_i\left(k\right)\Lambda_i\, G\left(p-k\right) U\ G\left(p-k\right)\Lambda_i, \tag{9}$$

where an index i indicates σ, ω, π, ρ and other mesons, D_i (the dashed line) is the corresponding meson propagator and Λ_i is the bare nucleon-meson vertex. Substituting an iteration expansion of Eq. (6) into Eq. (9), $\Gamma(p)$ is expanded as

$$\Gamma(p) = \sum_{n=1}^{\infty} n\,\Gamma_{(n)}(p), \tag{10}$$

$$\begin{aligned}\Gamma_{(n)}(p) &= \sum_i \int \frac{d^4k}{(2\pi)^4} D_i(k)\Lambda_i\, G^{(0)}(p-k)\overset{1}{U}\, G^{(0)}(p-k)\overset{2}{U}\cdots \\ &\quad \times G^{(0)}(p-k)\overset{n}{U}\, G^{(0)}(p-k)\,\Lambda_i. \end{aligned}\tag{11}$$

Using identities [14]

$$\left(G^{(0)}(p)\right)^2 = -\frac{\partial}{\partial \not{p}} G^{(0)}(p), \tag{12}$$

$$G^{(0)}(p)\,\gamma^0\, G^{(0)}(p) = -\frac{\partial}{\partial p_0} G^{(0)}(p), \tag{13}$$

we have

$$\left(S\frac{\partial}{\partial \not{p}} + V\frac{\partial}{\partial p_0}\right) G^{(0)}(p) = -\,G^{(0)}(p)\,U G^{(0)}(p). \tag{14}$$

Thus Eq. (11) is rewritten as

$$\Gamma_{(n)}(p) = \frac{(-1)^n}{n!}\left(S\frac{\partial}{\partial \not{p}} + V\frac{\partial}{\partial p_0}\right)^n \Sigma(p), \tag{15}$$

where

$$\begin{aligned}\Sigma(p) &= \sum_i \int \frac{d^4k}{(2\pi)^4} D_i(k)\,\Lambda_i\, G^{(0)}(p-k)\,\Lambda_i, &(16)\\ &= \Sigma_F(p) + \Sigma_D(p). &(17)\end{aligned}$$

$\Sigma_D(p)$ is the Fock potential term and yields momentum dependence in the potential.

Because our classical potential U does not depend on momentum p, the contribution of $\Sigma_D(p)$ is neglected in the following investigation of this work. Then we replace $\Sigma(p)$ in Eq. (15) by $\Sigma_F(p)$ and renormalize it. Consequently, Eq. (15) reduces to

$$\Gamma_{(n)}(p) = \frac{(-1)^n}{n!}\left(S\frac{\partial}{\partial \not{p}} + V\frac{\partial}{\partial p_0}\right)^n \Sigma_F^R(p). \tag{18}$$

$\Sigma_F^R(p)$ is the renormalized self-energy of a nucleon in the free space and is just the meson cloud itself. Therefore the correction $\Gamma(p)$ to the potential U is the effect of the meson cloud. Here it is noted that we do not take into account the modification of the meson cloud due to nuclear medium. It will be considered in the next section.

The relation (18) between the quantum correction to the potential and the self-energy of nucleon is similar to Ward identity in QED. Due to mass renormalization

$$\Sigma_F^R(p)\Big|_{\not p=M}=0, \tag{19}$$

and wave-function renormalization

$$\frac{\partial}{\partial \not p}\Sigma_F^R(p)\Bigg|_{\not p=M}=0, \tag{20}$$

$\Sigma_F^R(p)$ is written around $\not p=M$ as

$$\Sigma_F^R(p)=\frac{1}{2}\left(\frac{\not p-M}{M}\right)^2\zeta_{(2)}+\frac{1}{3!}\left(\frac{\not p-M}{M}\right)^3\zeta_{(3)}+\frac{1}{4!}\left(\frac{\not p-M}{M}\right)^4\zeta_{(4)}+\cdots. \tag{21}$$

Substituting this expansion into Eq. (18) and after rearrangement,

$$\begin{aligned}\Gamma(p) \;=\;& \bar U\left(\bar U\bar\zeta_{(2)}-\frac{1}{2}\bar U^2\bar\zeta_{(3)}+\frac{1}{3!}\bar U^3\bar\zeta_{(4)}+\cdots\right)\\ &-\frac{1}{2}\left[\bar U(\not p-M)+(\not p-M)\bar U\right]\bar\zeta_{(2)}\\ &+\frac{2}{3!}\left[\bar U^2(\not p-M)+\bar U(\not p-M)\bar U+(\not p-M)\bar U^2\right]\bar\zeta_{(3)}\\ &-\frac{3}{4!}\left[\bar U^3(\not p-M)+\bar U^2(\not p-M)\bar U+\bar U(\not p-M)\bar U^2+(\not p-M)\bar U^3\right]\bar\zeta_{(4)}\\ &+\cdots,\end{aligned} \tag{22}$$

where

$$U=\bar U M, \tag{23}$$

$$\zeta_{(n)}=\bar\zeta_{(n)}M. \tag{24}$$

Now we take the two serious assumptions that the replacement,

$$\bar\zeta_{(2)}\to\xi,\quad \bar\zeta_{(3)}\to\xi^2,\quad \bar\zeta_{(4)}\to\xi^3,\quad \cdots, \tag{25}$$

is possible in Eq. (22) and

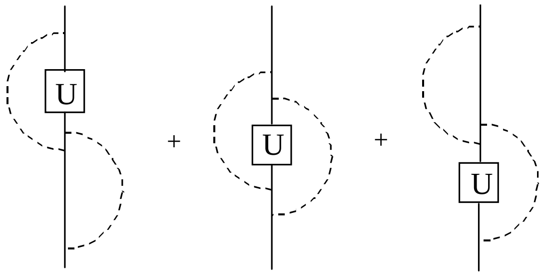
$$\left|\bar U\xi\right|\ll 1 \tag{26}$$

is satisfied. We expect that not all the detailed information of nucleon is necessary to describe nucleus or nuclear matter. An assumption (25) is thus adopted to express the effect of meson cloud by single quantity. In the next section, ξ is related to the isoscalar anomalous magnetic moment of nucleon. The propriety of Eq. (26) will be decided by the result of the calculation of nuclear matter saturation property. Under these assumptions, the total potential is given by

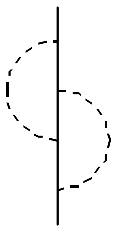
$$U_{tot} = U + \Gamma(p), \tag{27}$$

$$= U\left(1+\xi\bar{U}\right) - \frac{1}{2}\left[\bar{U}\left(\not{p}-M\right)+\left(\not{p}-M\right)\bar{U}\right]\xi. \tag{28}$$

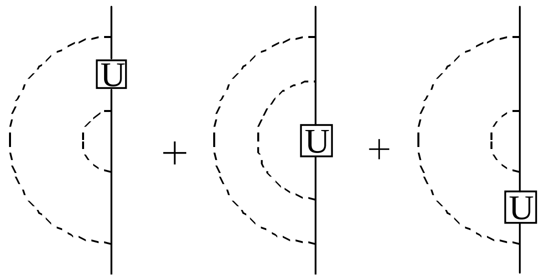
So far we have studied a simplest first-order correction (9) to the potential. However the result (28) is valid even for higher-order corrections. The reason is the same as the case of Ward identity in QED. It is easily seen that the two-loop correction (the solid line indicates $G(p)$ of Eq. (6))



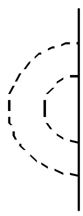
is derived from the two-loop self-energy (the solid line indicates $G_F^{(0)}(p)$ of Eq. (8))



using the Ward-like equation (18). Similarly, another two-loop correction

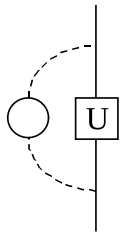


is derived from the self-energy,



The renormalization conditions (19) and (20) and so the expression of the self-energy (21) do not depend on the perturbation expansion. Thus, if the Ward-like equation (18) and the assumptions (25) and (26) are satisfied, Eq. (28) is also valid even for any higher-order corrections.

However it is noted that Eq. (15) is true but Eq. (18) is an approximation, which is not precise when the internal nucleon loop graphs are taken into account. For an example, the following correction to the potential is considered:



This includes the meson self-energy $\Pi_i\,(q)$. In nuclear medium, it is given by

$$\Pi_{\,i}\,(q) = \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\,[\,G\,(k)\,\Lambda_i\,G\,(k-q)\,\Lambda_i\,]. \tag{29}$$

Using an iteration expansion of Dyson equation (6), we have

$$\Pi_{\,i}\,(q) = \Pi_i^{(0)}\,(q) + \Pi_i^{(1)}\,(q) + \Pi_i^{(2)}\,(q) + \cdots\,, \tag{30}$$

where

$$\Pi_i^{(0)}\,(q) = \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\big[\,G^{(0)}\,(k)\,\Lambda_i\,G^{(0)}\,(k-q)\,\Lambda_i\,\big], \tag{31}$$

$$\begin{aligned} \Pi_i^{(1)}\,(q) \;\; = \;\; & \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\big[\,G^{(0)}\,(k)\,UG^{(0)}\,(k)\,\Lambda_i\,G^{(0)}\,(k-q)\,\Lambda_i \\ & +\; G^{(0)}\,(k)\,\Lambda_i\,G^{(0)}\,(k-q)\,U\,G^{(0)}\,(k-q)\,\Lambda_i\,\big]\,. \end{aligned} \tag{32}$$

The contributions of $\Pi_i^{(n)}\,(q) \quad (n = 1,\,2,\,3,\,\cdots)$ are just the effects of nuclear medium. Thus $\Sigma\,(p)$ in Eq. (15) cannot be replaced by the free self-energy $\Sigma_F\,(p)$ and so Eq. (18) is no longer valid. If $\Pi_{\,i}\,(q)$ is replaced by the meson self-energy in the free space

$$\Pi_i^F\,(q) = \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big[\,G_F^{(0)}\,(k)\,\Lambda_i\,G_F^{(0)}\,(k-q)\,\Lambda_i\,\Big], \tag{33}$$

and is further renormalized, then the Ward-like equation (18) is recovered. We note that the differences between $\Pi_{\,i}\,(q)$ and $\Pi_i^F\,(q)$ bring the modification of meson cloud of nuclear nucleon compared with free nucleon.

Because U_{tot} of Eq. (28) includes quantum corrections, the Lagrangian of the nucleon in symmetric nuclear matter is

$$\mathcal{L}_N = \bar{\psi}_{(0)} \left(\not{p} - M - U_{tot}\right) \psi_{(0)}, \tag{34}$$

$$= \bar{\psi}_{(0)} \left[\left(\not{p} - M\right) + \frac{1}{2}\xi\,\bar{U}\left(\not{p} - M\right) + \frac{1}{2}\xi\left(\not{p} - M\right)\bar{U} - \left(1 + \xi\,\bar{U}\right)U\right]\psi_{(0)}. \tag{35}$$

Assuming Eq. (26), this becomes

$$\mathcal{L}_N = \bar{\psi}_{(0)} \left[\left(1 + \frac{1}{2}\xi\,\bar{U}\right)\left(\not{p} - M\right)\left(1 + \frac{1}{2}\xi\,\bar{U}\right) - \left(1 + \xi\,\bar{U}\right)U\right]\psi_{(0)}, \tag{36}$$

$$= \bar{\psi}_{(0)} \left[\left(1 + \xi\,\bar{U}\right)^{1/2}\left(\not{p} - M\right)\left(1 + \xi\,\bar{U}\right)^{1/2} - \left(1 + \xi\,\bar{U}\right)U\right]\psi_{(0)}, \tag{37}$$

$$= \bar{\psi}_{(0)} \left[Z_2^{-1/2}\left(\not{p} - M\right)Z_2^{-1/2} - Z_2^{-1}U\right]\psi_{(0)}, \tag{38}$$

where

$$Z_2^{-1} = 1 + \xi\,\bar{U}. \tag{39}$$

Introducing a renormalized nucleon wave function ψ_R as

$$\psi_{(0)} = Z_2^{1/2}\,\psi_R, \tag{40}$$

the Lagrangian is written by

$$\mathcal{L}_N = \bar{\psi}_R\left(\not{p} - M - \ U\right)\psi_R. \tag{41}$$

This is consistent to the Dyson equation (6) assumed first and is the Lagrangian for the dressed wave function, which is not a bare nucleon but includes the meson cloud.

3 Determination of renormalization constant

In the previous section, we employed an assumption (25) and introduced the quantity ξ. Here we determine it from the isoscalar anomalous magnetic moment of a free nucleon and further modify it to take into account the effect of nuclear medium on the nucleon. The renormalized self-energy of a free nucleon is generally given by

$$\Sigma_F^R(p) = \left(\not{p} - M\right)a\left(p^2/M^2\right) + M\,b\left(p^2/M^2\right), \tag{42}$$

where a and b are functions of p^2/M^2. The mass renormalization condition (19) becomes

$$b\,(1) = 0, \tag{43}$$

and the wave function renormalization condition (20) is

$$a\,(1) + 2\,b'\,(1) = 0. \tag{44}$$

The previously used self-energy (21) has the same form as Eq. (42) if $a\left(p^2/M^2\right)$ and $b\,(p^2/M^2)$ are expanded around $p^2 = M^2$;

$$a\left(\frac{p^2}{M^2}\right) = a_0 + \frac{p^2 - M^2}{M^2}\,a_1 + \cdots\,, \tag{45}$$

$$b\left(\frac{p^2}{M^2}\right) = b_0 + \frac{p^2 - M^2}{M^2}\,b_1 + \cdots\,. \tag{46}$$

Using an identity

$$\left(\not{p} - M\right)^2 = \left(p^2 - M^2\right) - 2M\left(\not{p} - M\right), \tag{47}$$

the expansion coefficients in Eq. (45) are given by

$$a_0 = \frac{1}{2}\,(-2)\,\bar{\zeta}_{(2)} + \frac{1}{3!}\,(-2)^2\,\bar{\zeta}_{(3)} + \frac{1}{4!}\,(-2)^3\,\bar{\zeta}_{(4)} + \frac{1}{5!}\,(-2)^4\,\bar{\zeta}_{(5)} + \cdots\,, \tag{48}$$

$$a_1 = \frac{1}{3!}\,\bar{\zeta}_{(3)} + \frac{2\,(-2)}{4!}\,\bar{\zeta}_{(4)} + \frac{3\,(-2)^2}{5!}\,\bar{\zeta}_{(5)} + \frac{4\,(-2)^3}{6!}\,\bar{\zeta}_{(6)} + \cdots\,. \tag{49}$$

On the other hand, the coefficients in Eq. (46) are given by the renormalization conditions,

$$b_0 = 0, \tag{50}$$

$$a_0 + 2\,b_1 = 0. \tag{51}$$

Here, assuming Eq. (25), Eq. (48) becomes

$$\exp\left(-2\,\xi\right) + 2\,(1 + a_0)\,\xi = 1. \tag{52}$$

If $|\xi| \ll 1$, this has a trivial solution $\xi = 0$ and a nontrivial one,

$$\xi \approx -\,a_0 = -\,a\left(p^2/M^2 = 1\right). \tag{53}$$

Therefore, once a value of a_0 is given, the value of ξ is determined.

Then we determine a_0 by means of the Ward-Takahashi identity for the isoscalar current Υ^μ_R of a free nucleon,

$$(p' - p)_\mu\,\Upsilon^\mu_R\,(p', p) = G^R_F\,(p')^{-1} - G^R_F\,(p)^{-1}\,, \tag{54}$$

where

$$G^R_F\,(p)^{-1} = \not{p} - M - \Sigma^R_F\,(p)\,. \tag{55}$$

Differentiating Eq. (54) by p_μ and p'_μ and then taking the difference between them, we have

$$\Upsilon^{\mu}_{R}\left(p',p\right)+\frac{1}{2}\,q_{\nu}\left(\frac{\partial}{\partial p'_{\mu}}-\frac{\partial}{\partial p_{\mu}}\right)\Upsilon^{\mu}_{R}\left(p',p\right)=\gamma^{\mu}-\frac{1}{2}\frac{\partial}{\partial p'_{\mu}}\Sigma^{R}_{F}\left(p'\right)-\frac{1}{2}\frac{\partial}{\partial\,p_{\mu}}\Sigma^{R}_{F}\left(p\right), \tag{56}$$

where $q_{\mu}=p'_{\mu}-p_{\mu}$. From Eq. (42),

$$\left(\partial/\partial\,p_{\mu}\right)\Sigma^{R}_{F}\left(p\right)=\gamma^{\mu}\,a\left(\hat{p}^{2}\right)+2\left(\not{p}-M\right)p^{\mu}a'\left(\hat{p}^{2}\right)/M^{2}+2\,p^{\mu}\,b'\left(\hat{p}^{2}\right)/M, \tag{57}$$

where $\hat{p}^{2}=p^{2}/M^{2}$. On the mass shell, this becomes

$$\left(\partial/\partial\,p_{\mu}\right)\Sigma^{R}_{F}\left(p\right)=\gamma^{\mu}a\left(1\right)+2\,p^{\mu}\,b'\left(1\right)/M. \tag{58}$$

Then, using the renormalization condition (51), the rhs of Eq. (56) becomes

$$\text{rhs of Eq. (56)}=\left(1-a\left(1\right)\right)\gamma^{\mu}+\frac{1}{2M}\left(p^{\mu}+p'^{\mu}\right)a\left(1\right). \tag{59}$$

On the other hand, we use the phenomenological current for the LHS of Eq. (56):

$$\Upsilon^{\mu}_{R}\left(p',p\right)=F_{1}\left(q^{2}\right)\gamma^{\mu}+iF_{2}\left(q^{2}\right)\sigma^{\mu\nu}q_{\nu}/\left(2M\right). \tag{60}$$

Using the Gordon decomposition, and under an approximation $\left|q^{2}/M^{2}\right|\ll 1$, this becomes

$$\Upsilon^{\mu}_{R}\left(p',p\right)\simeq\left(1+F_{2}\left(0\right)\right)\gamma^{\mu}-\frac{1}{2M}\left(p^{\mu}+p'^{\mu}\right)F_{2}\left(0\right). \tag{61}$$

Therefore the second term in the LHS of Eq. (56) vanishes. Comparing between Eqs. (59) and (61), we obtain

$$a\left(1\right)=a_{0}=-F_{2}\left(0\right)=-\left(\Delta\mu_{p}+\Delta\mu_{n}\right)=0.12, \tag{62}$$

where $\Delta\mu_{p(n)}$ is the anomalous magnetic moment of a proton (neutron). As a result, Eq. (53) leads to

$$\xi\approx\Delta\mu_{p}+\Delta\mu_{n}=-\,0.12. \tag{63}$$

Next we have to consider the effect of nuclear medium on the meson cloud. Equation (63) suggests that this can be done by the following replacement,

$$\xi\rightarrow\xi^{*}\approx\Delta\mu^{*}_{p}+\Delta\mu^{*}_{n}, \tag{64}$$

where $\Delta\mu^{*}_{p(n)}$ is the anomalous magnetic moment of the nuclear nucleon. So as to estimate this quantity, we remember that the magnetic moment of a free nucleon can be simply explained by the constituent quark picture. The mass of the nuclear nucleon is reduced by the scalar potential S,

$$M^* = M + S = m^* M, \tag{65}$$

and thus the masses of its constituent u and d quarks are

$$M^*_{u(d)} = m^* M_{u(d)}. \tag{66}$$

This means that the magnetic moment of a nuclear nucleon is

$$\mu^*_{p(n)} = \mu_{p(n)}/m^*. \tag{67}$$

Because its Dirac part is given by

$$\mu^*_D = \frac{1}{2}\,(1+\tau_3)\,\frac{e}{2M^*} = \frac{\mu_D}{m^*}, \tag{68}$$

its anomalous part also becomes

$$\Delta\mu^*_{p(n)} = \Delta\mu_{p(n)}/m^*. \tag{69}$$

Consequently, the precise value of ξ^* is a nontrivial solution ($\neq 0$) of the following equation as Eq. (52),

$$\exp\left(-2\,\xi^*\right) + 2\left[1 - \left(\Delta\mu^*_p + \Delta\mu^*_n\right)\right]\ \xi^* = 1. \tag{70}$$

In the next section, we will treat ξ^* as a parameter in solving the self-consistent equation of m^*. The value of ξ^* is determined to reproduce the nuclear matter saturation property. Then we will calculate

$$\Delta\mu^*_p + \Delta\mu^*_n = \frac{\exp\left(-2\,\xi^*\right) + 2\,\xi^* - 1}{2\xi^*}, \tag{71}$$

and compare

$$\Delta\mu_p + \Delta\mu_n = m^*\left(\Delta\mu^*_p + \Delta\mu^*_n\right) \tag{72}$$

with its experimental value -0.12.

Finally we want to mention the meaning of the approximation (25) again. It has ensured the convergence of the RHS of Eq. (48) and so has related ξ to a_0. We have only used this a_0 in Σ^R_F, which is related to the anomalous magnetic moment by means of Ward-Takahashi identity. Other quantities b_0 and b_1 in Σ^R_F disappear due to the renormalization conditions. The higher order quantities a_1, a_2, b_2, *etc.* are not concerned. Therefore we can see that the approximation (25) is just the prescription to ensure the convergence of the perturbation expansion and to extract the single physical quantity from Σ^R_F.

4 Modified Walecka model for dressed nucleon

Here, based of the results of the preceding sections, we will extend the Walecka model for symmetric nuclear matter to take into account the effect of the meson cloud. In the Walecka model, the Lagrangian is for a bare nucleon expressed by an unrenormalized wave function $\psi_{(0)}$,

$$\mathcal{L}_N^W = \bar{\psi}_{(0)} \left(\not{p} - M - S_0 - \gamma^0 V_0 \right) \psi_{(0)}. \tag{73}$$

The scalar and vector potentials are given by the mean fields of the σ and ω mesons:

$$S_0 = -g_{NN\sigma} \left\langle \sigma \right\rangle_0 , \tag{74}$$

$$V_0 = g_{NN\omega} \left\langle \omega_0 \right\rangle_0 , \tag{75}$$

where $g_{NN\sigma(\omega)}$ is the nucleon $\sigma\,(\omega)$-meson coupling constant. However our desired Lagrangian should be for the dressed nucleon surrounded by meson cloud. It is expressed by a renormalized wave function ψ_R as (See Eq. (41).)

$$\mathcal{L}_N^R = \bar{\psi}_R \left(\not{p} - M - S - \gamma^0 V \right) \psi_R. \tag{76}$$

In order to introduce the renormalized wave function ψ_R in place of the unrenormalized one $\psi_{(0)}$, we should remember Eq. (38). In analogy to the counter term contribution to the wave function renormalization, we modify the Walecka Lagrangian (73) as follows:

$$\not{p} - M \to {Z_2}^{-1/2} \left(\not{p} - M \right) {Z_2}^{-1/2}. \tag{77}$$

Then, using Eq. (40), the renormalized Walecka Lagrangian is

$$\mathcal{L}_N^{RW} = \bar{\psi}_{(0)} \left[{Z_2}^{-1/2} \left(\not{p} - M \right) {Z_2}^{-1/2} - S_0 - \gamma^0 V_0 \right] \psi_{(0)}, \tag{78}$$

$$= \bar{\psi}_R \left[\not{p} - M - Z_2 \left(S_0 + \gamma^0 V_0 \right) \right] \psi_R. \tag{79}$$

Defining the renormalized potential by

$$S + \gamma^0 V = Z_2 \left(S_0 + \gamma^0 V_0 \right) , \tag{80}$$

Eq. (79) can be regarded as our expected Lagrangian (76). The renormalization constant Z_2 is given by Eq. (39),

$$Z_2^{-1} = 1 - \eta \left(\bar{s} + \gamma^0 \bar{v} \right) , \tag{81}$$

where $S = \bar{s}M$, $V = \bar{v}M$ and $\eta = -\xi^*$. (The quantity ξ in Eq. (39) is replaced by

ξ^* according to Eq. (64).) Although ξ^* is determined by Eq. (70), we will treat η as a parameter in the following. Substituting Eq. (81) into Eq. (80), we have

$$\bar{s} = \bar{s}_0 + \eta \left(\bar{s}^2 + \bar{v}^2\right), \tag{82}$$

$$\bar{v} = \bar{v}_0 + 2\,\eta\,\bar{s}\,\bar{v}, \tag{83}$$

where $S_0 = \bar{s}_0 M$ and $V_0 = \bar{v}_0 M$. Up to the second order of $\bar{s}_0 = -\,\bar{\sigma}$ and $\bar{v}_0 = \bar{\omega}_0$, these are

$$\bar{s} = -\,\bar{\sigma} + \eta \left(\bar{\sigma}^2 + \bar{\omega}_0^2\right), \tag{84}$$

$$\bar{v} = \bar{\omega}_0 - 2\,\eta\,\bar{\sigma}\,\bar{\omega}_0. \tag{85}$$

The second and third term of Eq. (84) and the second term of Eq. (85) are just *the scalar polarizability*, *vector polarizability* and *the mixed scalar-vector polarizability* in Eqs. (1) and (2), respectively. (In the present theory, the term proportional to $|\mathbf{p}|^2 \langle\sigma\rangle^2$ in Eq. (2) comes from the so-called Z-graph contribution [15] and the polarizability ξ_p does not appear.)

Since

$$|\eta\,\bar{s}| \ll 1, \quad |\eta\,\bar{v}| \ll 1, \tag{86}$$

have been assumed in Eq. (26), the relations

$$\left|\eta\,\bar{s}^2\right| \ll 1, \quad \left|\eta\,\bar{v}^2\right| \ll 1, \tag{87}$$

are also satisfied. Then we assume $\eta\,\bar{v}^2 \approx \eta\,\bar{s}^2$ or approximate Eq. (82) as

$$\bar{s} = \bar{s}_0 + 2\,\eta\,\bar{s}^2. \tag{88}$$

As a result the vector polarizability seems to disappear. Equation (88) is rewritten as

$$S = \frac{S_0}{1 + \lambda\,(1 - m^*)}, \tag{89}$$

where $\lambda = 2\,\eta$ and Eq. (65) has been used. According to Eq. (74) we can define the renormalized nucleon σ-meson coupling constant by

$$S = -g^R_{NN\sigma}\,\langle\sigma\rangle_0\,. \tag{90}$$

It is given by

$$g^R_{NN\sigma} = \frac{g_{NN\sigma}}{1 + \lambda\,(1 - m^*)}, \tag{91}$$

$$\approx [(1 - \lambda) + \lambda\,m^*]\,g_{NN\sigma}. \tag{92}$$

The result (92) is just the hybrid derivative scalar coupling of Ref. [11]. Similarly we define the renormalized nucleon ω-meson coupling constant $g_{NN\omega}^R$ by

$$V=\frac{V_0}{1+\lambda\left(1-m^*\right)}=g_{NN\omega}^R\left\langle\omega_0\right\rangle_0, \tag{93}$$

$$g_{NN\omega}^R = \frac{g_{NN\omega}}{1+\lambda\left(1-m^*\right)}, \tag{94}$$

$$\approx \left[(1-\lambda)+\lambda\, m^*\right] g_{NN\omega}. \tag{95}$$

Therefore a relation

$$\frac{g_{NN\sigma}^R}{g_{NN\sigma}}=\frac{g_{NN\omega}^R}{g_{NN\omega}}, \tag{96}$$

is satisfied. The Walecka model corresponds to the case of $\lambda=0$. On the other hand, the DSC model corresponds to the case in which the scalar coupling is renormalized by Eq. (92) with $\lambda=1$ but the vector coupling is not renormalized ($\lambda=0$ in Eq. (95)). Thus the relation (96) is not satisfied and this is one of the reasons of the failure of the DSC model.

Consequently, our model Lagrangian has the same form as the Walecka Lagrangian except for the renormalized nucleon σ (ω)-meson coupling constant (92)((95)):

$$\mathcal{L}^{RW}=\bar{\psi}\left(\not{p}-M\right)\psi-\frac{1}{2}m_\sigma^2\left\langle\sigma\right\rangle^2+\frac{1}{2}m_\omega^2\left\langle\omega_0\right\rangle^2+g_{NN\sigma}^R\left\langle\sigma\right\rangle\bar{\psi}\psi-g_{NN\omega}^R\left\langle\omega_0\right\rangle\bar{\psi}\gamma^0\psi, \tag{97}$$

where subscripts R and 0 are omitted. The σ and ω mean-field, $\langle\sigma\rangle$ and $\langle\omega_0\rangle$, are expressed in terms of m^* and v through Eqs. (90) and (93). Then we have the energy-per-particle W for symmetric nuclear matter written in units of M as

$$\frac{W}{M}=\frac{\langle E_k^*\rangle}{M}+\frac{1}{2C_s\hat{\rho}}\left[\frac{1-m^*}{1-\lambda\left(1-m^*\right)}\right]^2+\frac{1}{2}C_v\left[1-\lambda\left(1-m^*\right)\right]^2\hat{\rho}-1, \tag{98}$$

where $\langle E_k^*\rangle$ is the average kinetic energy, $\hat{\rho}=\rho_v/\rho_v^{(0)}$ with the vector density ρ_v and the nuclear matter saturation density $\rho_v^{(0)}$. $C_{s(v)}$ are defined by

$$C_{s(v)}\equiv\frac{g_{NN\sigma(\omega)}^2\rho_v^{(0)}}{m_{\sigma(\omega)}^2 M}. \tag{99}$$

These are determined to fulfill the saturation condition,

$$\partial W/\partial\hat{\rho}\big|_{\hat{\rho}=1}=0, \tag{100}$$

$$W_0\equiv W\left(\hat{\rho}=1\right)=-15.75\,\mathrm{MeV}, \tag{101}$$

at the Fermi momentum $k_F = 1.30\,\mathrm{fm}^{-1}$. The results are

$$C_s = \left[\frac{1-m_0^*}{1-\lambda\,(1-m_0^*)}\right]^2 \Bigg/ \left[v_0 + \frac{1}{2}\left(\frac{E_F^*}{M} - m_0^*\frac{\rho_s^{(0)}}{\rho_v^{(0)}}\right)\right], \tag{102}$$

$$C_v = \frac{v_0}{\left[1-\lambda\,(1-m_0^*)\right]^2}, \tag{103}$$

where E_F^* is the Fermi surface energy of nuclear matter, $\rho_s^{(0)}$ is the scalar density at saturation, v_0 is given by

$$v_0 = 1 + W_0/M - E_F^*/M. \tag{104}$$

The renormalized mass m_0^* at saturation is determined by the self-consistency equation $\partial W/\partial m^*|_{\hat{\rho}=1} = 0$, which is written explicitly as

$$\frac{1}{2}\left(\frac{E_F^*}{M} - m_0^*\frac{\rho_s^{(0)}}{\rho_v^{(0)}}\right) + v_0 - (1-m_0^*)\left[1-\lambda\,(1-m_0^*)\right]\left[\frac{\rho_s^{(0)}}{\rho_v^{(0)}} + \frac{\lambda\, v_0}{1-\lambda\,(1-m_0^*)}\right] = 0. \tag{105}$$

5 Calculations and analyses

Here we calculate the properties of symmetric nuclear matter according to the previous section. First, Fig. 1 shows the effective mass m^* as functions of density. The renormalization constant λ is treated as a parameter. For example, we choose $\lambda = 0.0$, 0.35, 0.7 and 1.0. The result using $\lambda = 0$ corresponds to the Walecka model. The effective mass becomes larger as λ increases. As mentioned in Introduction, $m^* \approx 0.6$ is favorable at saturation density $\rho_v/\rho_v^{(0)} = 1$. Thus large value of $\lambda \geq 0.7$ seems to be not suitable. To see this more clearly, Fig. 2 shows the scalar potential at saturation as a function of λ. The value $0.3 \leq \lambda \leq 0.4$ seems to be appropriate for $S \approx -375\,\mathrm{MeV}$ corresponding to $m^* \approx 0.6$. Consequently the value of λ is severely restricted. This is because the scalar potential S is relatively sensitive to λ. The vector potential V is also sensitive to λ as shown in Fig. 3.

Next we calculate the energy-per-particle W (Eq. (98)) as functions of density in Fig. 4. It is seen that the slopes of the curves at $1 \leq \hat{\rho}$ becomes gentle as λ increases. This implies that the incompressibility becomes smaller as λ increases. Then, in Fig. 5, we show the nuclear matter incompressibility K_v as a function of λ. It is difficult to determine K_v uniquely from empirical data and so there is large uncertainty in its value in the literature [16]. Assuming $200\,\mathrm{MeV} \leq K_v \leq 300\,\mathrm{MeV}$, λ is restricted into the region $0.3 < \lambda < 0.7$. However this allowed range of λ is too large to determine it uniquely. On the other hand, Pearson [17] pointed out that the correlation between K_v and K_{Coul}, the Coulomb coefficient in the leptodermous expansion of the incompressibility, is rather uniquely determined from breathing-mode data. Thus we further calculate $K_v - K_{Coul}$ correlation in Fig. 6. The shaded area is the breathing-mode data in Ref. [17]. The circles indicate the results using $\lambda = 0.0 \sim 1.0$ and the triangle indicates the result of $\lambda = 0.35$. It is seen that only the results using $0.2 \le \lambda \le 0.35$ can cross the shaded band. Taking into account the above analyses of m^* and K_v, we can conclude that the renormalization constant is uniquely determined to $\lambda \approx 0.35$. It has also been found that the combination of K_v and $K_v - K_{Coul}$ correlation is quite useful to determine the value of λ. The obtained value prefers large incompressibility $K_v \approx 300\,\text{MeV}$. In the following table, we summarize the values of the scalar and vector coupling constants, $(g_{NN\sigma})^2/(4\pi)$ and $(g_{NN\omega})^2/(4\pi)$, the effective mass m^* at saturation, the scalar S and vector V potentials at saturation, the incompressibility K_v and the Coulomb coefficient K_{Coul} for various values of λ.

λ	$\frac{(g_{NN\sigma})^2}{4\,\pi}$	$\frac{(g_{NN\omega})^2}{4\,\pi}$	m^*	S (MeV)	V (MeV)	K_ν (MeV)	K_{Coul} (MeV)
0.0	9.76	15.1	0.541	− 431	354	547	−8.80
0.35	11.4	17.4	0.603	− 372	301	293	−5.32
0.7	12.3	17.9	0.667	− 312	246	193	−3.45
1.0	12.2	17.1	0.715	− 267	204	155	−2.68

Although λ has been treated as a parameter in the above calculations, it is related to the isoscalar anomalous magnetic moment of a free nucleon according to Eqs. (71) and (72). Thus we calculate the anomalous moment using $\lambda = 0.35$ and $m^* = 0.603$ that are most appropriate values to the saturation properties as shown above. If the experimental value is reproduced, λ $(= 2\,\eta = -\,2\,\xi^*)$ is not a phenomenological parameter but has a physical meaning. The result

$$\Delta\mu_p + \Delta\mu_n = -\,0.119 \tag{106}$$

remarkably agrees with the experimental value $-\,0.120$. This perfect agreement is somewhat accidental because of uncertainty of nuclear matter saturation properties. However it strongly supports that the effect of meson cloud is important to nuclear matter properties. Finally we have to estimate the assumption Eq. (26) (or Eq. (86)). Because

$$|\eta\,\bar{s}| = 0.175 \times 0.4 = 0.07, \tag{107}$$

our assumption is consistent.

6 Summary

We have developed the modified Walecka model with the renormalized $NN\sigma$ and $NN\omega$ coupling constants that incorporate the polarizabilities of nucleon by the mean meson fields. They are theoretically derived from the quantum corrections to the mean-fields, which are just the effects of meson cloud of nucleon. Why does the meson cloud affect nuclear matter properties? As expressed by the Ward-like equation (18), this is because it couples to the mean scalar and vector fields in nuclear medium. In this respect, Birse [6] suggested that the polarizability is the response of the nucleon's structure to pushing in the presence of the meson fields, based on a nontopological soliton model. Of course, such a coupling must cause the change of meson cloud itself in the medium. In order to estimate this change, we first related the wave function renormalization Z_2 to the isoscalar anomalous magnetic moment of the free nucleon using Ward-Takahashi identity. Then we determined that of nuclear nucleon using a naive quark picture. This prescription may suggest an essential limitation of the interacting meson-nucleon field theory in which the nucleon is treated as an elementary point particle. The efforts to develop the consistent theory of nuclear matter based on the quark model [18] are also necessary. However our numerical results reproduce nuclear matter saturation properties well and overcome all the problems in the previous models. It indicates that the effect of meson cloud is really important to nuclear matter saturation properties.

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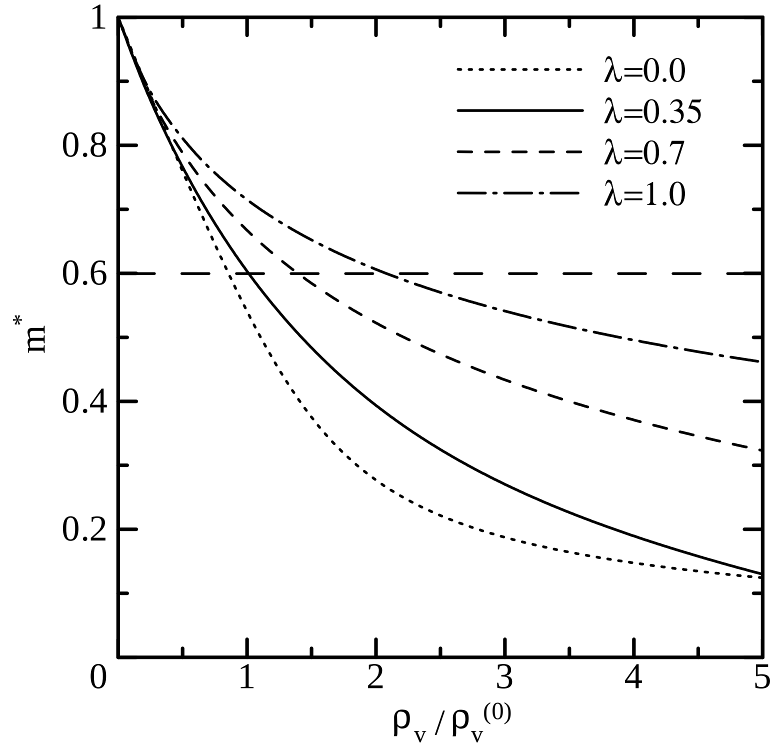

Figure 1: The density dependencies of the effective nucleon mass $m^* = M^*/M$. The dotted, solid, dashed and dot-dashed curves are the calculations using $\lambda = 0.0$, 0.35, 0.7 and 1.0 respectively.

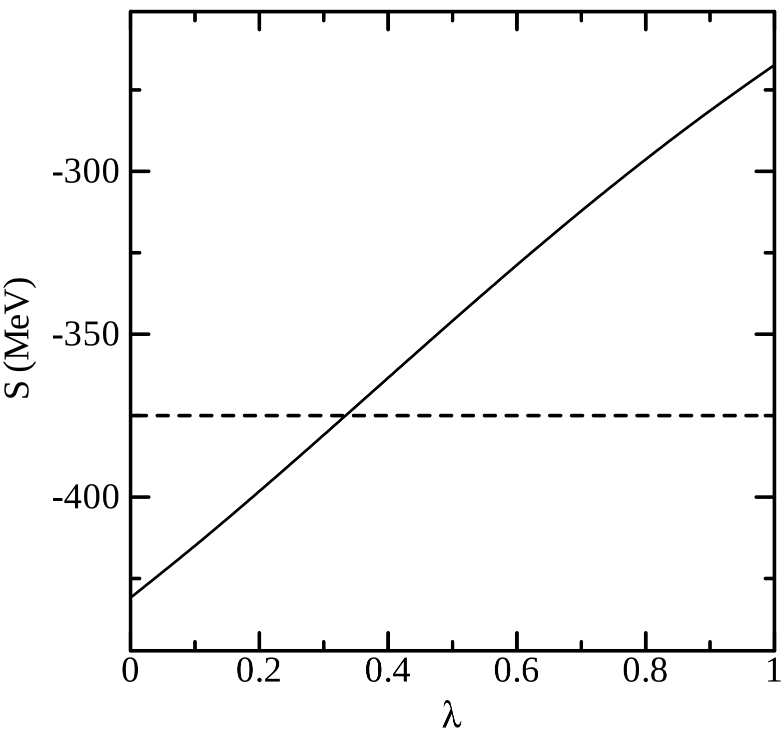

Figure 2: The scalar potential S at saturation density as a function of λ. The dashed line indicates $S = -375\,\text{MeV}$ corresponding to $m^* = 0.6$.

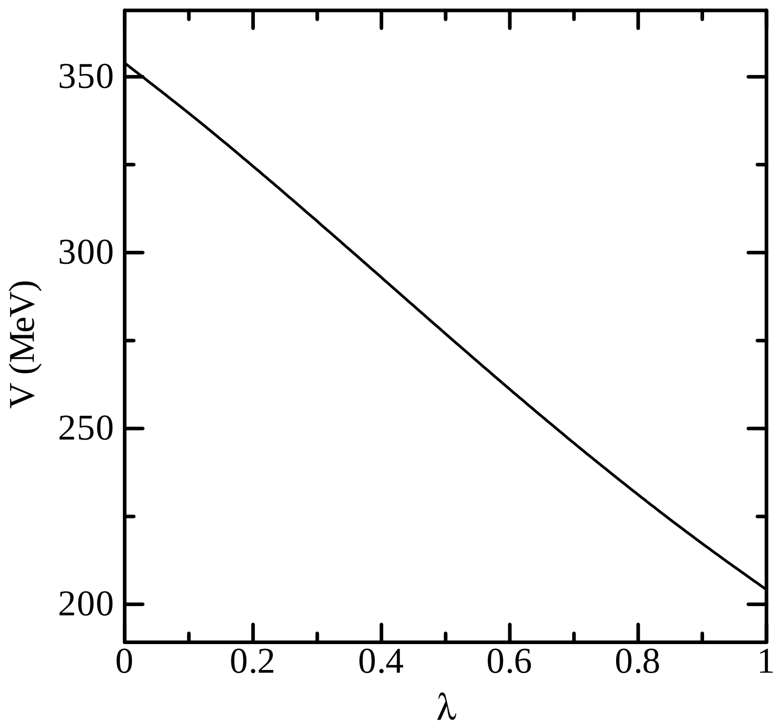

Figure 3: The same as Fig. 2 but for the vector potential V.

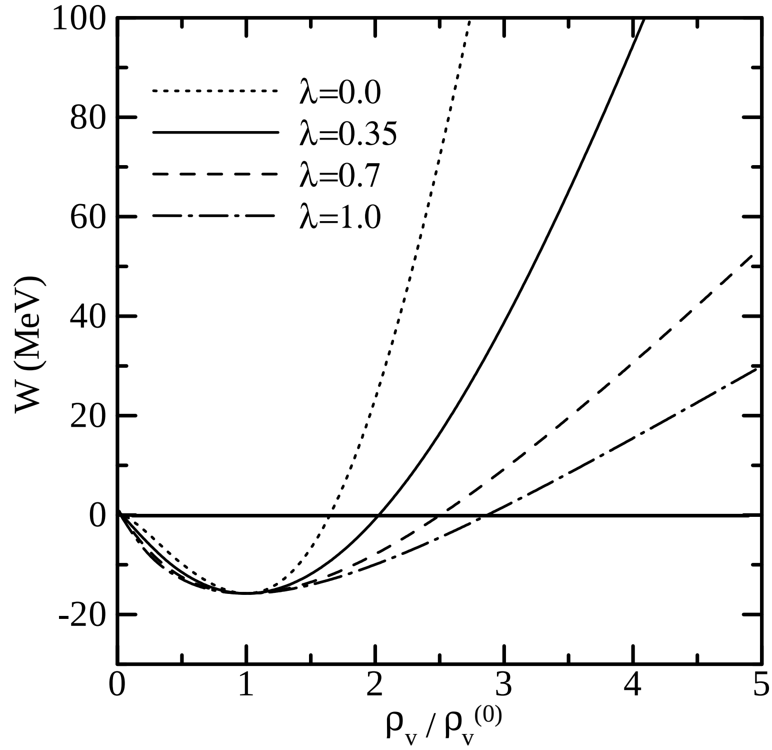

Figure 4: The same as Fig. 1 but for the energy-per-particle W.

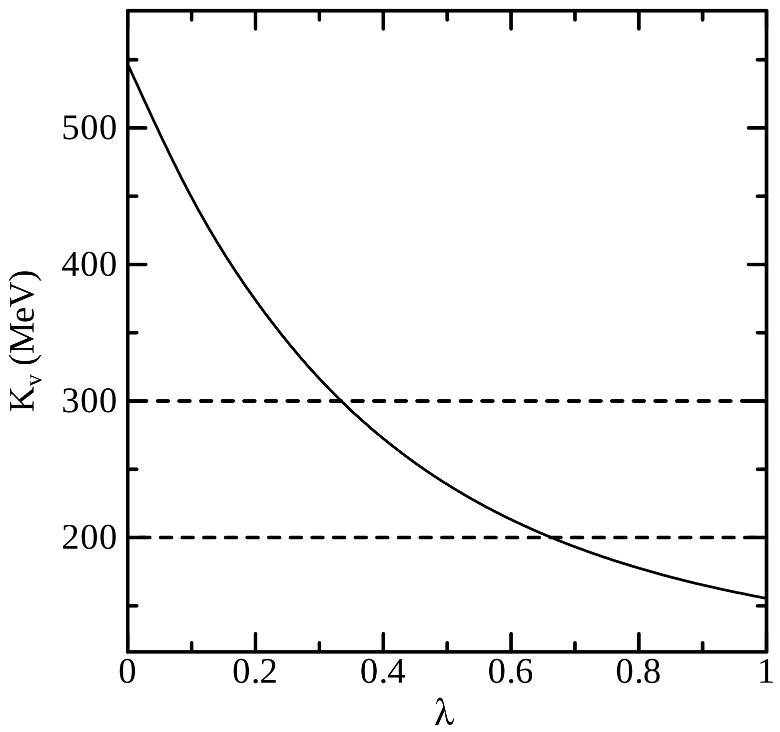

Figure 5: The same as Fig. 2 but for the incompressibility K_v.

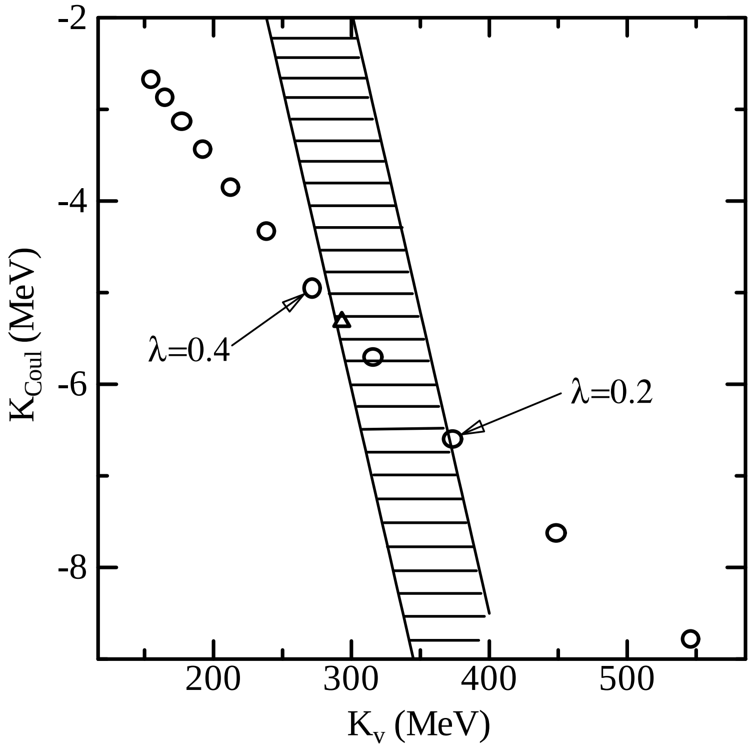

Figure 6: The $K_v - K_{Coul}$ correlation. The shaded band is the breathing-mode data in Ref. [17]. The open circles are the calculational results using $\lambda = 0.0 \sim 1.0$ and the triangle is the result using $\lambda = 0.35$.