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An Analytical Overview of Gravitational Waves

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ABSTRACT

The September 14, 2015 gravitational wave observations showed the inspiral of two black holes observed from Hanford and Livingston LIGO observatories. This detection was significant for two reasons: firstly, it coupled the result and avoided the possibility of a *false alarm* by 5σ , meaning that the detected "noise" was indeed from an astronomical source of gravitational waves. We will discuss the primary landscape of gravitational waves, their mathematical structure and how they can be used to predict the masses of the merger system. We will also discuss gravitational wave detector optimisations, and then we will consider the results from the detected merger GW150914.

We will consider a straight-forward mathematical approach, and we will primarily be interested in the mathematical modelling of gravitational waves from General Relativity (Section 1). We will first consider a "perturbed" Minkowski metric, and then we will discuss the properties of the perturbation addition tensor. We will then discuss on the gravitational field tensor, and how it arises from the perturbation tensor. We will then talk about the gauge condition, essentially the gauge "freedom" , and then we will talk about the curvature tensor, leading eventually to the effect of gravitational waves on a ring of particles. We will consider the *polarisation tensor*, which maps the amplitude and polarisation details. The polarisation splits into *plus polarised* and *cross polarised waves*, which is technically the effect of a propagating gravitational wave through a ring of particles. We will then talk about the *linearized* Einstein Field Equations, and how the physical system of merger is encoded into the mathematical structural unity of the metric.

We will then talk about the detection of these gravitational waves and how the detector can be *optimised*, or how the detector can be set so that any "noise" detected can fall in the error margins, and how the detector can prevent the interferometric "photon-noise" from being detected (Section 2.2). Then, we will discuss data results from the source GW150914 detection by LIGO (Section 3)

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1. GRAVITATIONAL WAVES AND THEIR MATHEMATICAL FORMULATION

1.1 GRAVITATIONAL WAVES

Gravitational waves are of extraordinary interest, because they are intrinsic results of General Relativity, and therefore the detection of these waves at LIGO is a proof of the mathematical stability of General Relativity. The basis of gravitational waves is by presenting a *Linearized* set of Einstein Equations that show a wave propagating through space-time. By considering perturbations around the usual Minkowski metric, we eventually write down the exact form of the Field equations. By deriving the Riemann tensor and the Ricci tensor in terms of the quantity $h_{\mu\nu}$, we will then talk about the gauge freedom and further consider the derived Field Equations. We then discuss the *Hilbert* and the *De Donder/Lorenz* gauge property, and then we will derive the wave-propagation equation in terms of the linear perturbations we have set for modelling gravitational waves. By considering the wave equation, which is essentially the D'Alembertian, we will write down the polarisations of the wave and its effect on a ring of particles, which are either *Plus polarised* or *Cross polarised*.

The considerations of gravitational waves arises from the perturbation of the *flat* Minkowski metric. If we say that the addition term is not small enough, then we are preventing the metric from being linear. Therefore, if we consider a *perturbed* Minkowski metric[2], $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$

Here, we are considering that the transformation property of the extra term $h_{\mu\nu}$ is defined by the usual transformation property as seen in tensor analysis. We can present a set of Linearized Einstein Field Equations in terms of a wave propagating through space-time. If we were to write down the Christoffel symbols, by considering the invariant nature of the Minkowski metric under coordinate transformations, we can derive the following result (1.1.1)

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2}\eta^{\sigma\lambda}\left(\partial_\mu h_{\nu\lambda} + \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}\right)$$

The Christoffel symbols can be collectivised to obtain the curvature, which is described by the Riemann tensor. We can use that result to find the Ricci tensor, which is given by the contracting of two indices in the original Riemann tensor. The following two equations depict the Riemann tensor and the Ricci tensor respectively: (1.1.2)

$$R_{\mu\nu\sigma\rho} = \eta_{\mu\lambda}\partial_\sigma\Gamma^{\lambda}_{\nu\rho} - \eta_{\mu\lambda}\partial_\rho\Gamma^{\lambda}_{\nu\sigma}$$

(This can further be expressed in terms of the addition term $h_{\mu\nu}$ which can be simply derived by juggling the respective indices and by substituting for the values as given by the already derived Christoffel symbols).

By contracting the indices ρ and σ , we can further get the Ricci tensor, (1.1.3)

$$R_{\mu\nu} = \frac{1}{2}\left(\partial_\rho\partial_\nu h^{\rho}_{\mu} + \partial_\rho\partial_\mu h^{\rho}_{\nu} - \partial_\gamma\partial^\gamma h_{\mu\nu}\right)$$

Here, $\partial_\gamma\partial^\gamma$ is the wave operator, which is sometimes shown in here[3] as $\Box$. We can write out the quantity $h_{\mu\nu}$ as $h = \eta^{\mu\nu}h_{\mu\nu}$, and the Ricci scalar, which is defined as $R = g^{\mu\nu}R_{\mu\nu}$ can be written in terms of the wave operator as the quantity (1.1.4)

$$R = \partial_\mu\partial_\nu h^{\mu\nu} - \Box h$$

We will want to define a T-R[4] perturbation quantity $\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^{\sigma}_{\sigma}$. We will find this term to be of significant importance later in the text. This is specifically referred to as the gravitational field tensor.

Specifically, we write the *Hilbert gauge condition.* This is analogous to the Lorentz gauge condition.

The most elementary solution to the wave equation discussed above is with the components written in terms of the amplitude, the exponential frequency, and the direction of propagation. The result is a *planar* wave solution, which we will list out as (1.1.5)

$$\hat{h}_{\mu\nu} = \Theta^{\mu\nu} + e^{i\psi_\lambda x^\lambda}$$

[2]Since we are preventing any higher-order terms from appearing in our linearized theory, we will want to define the nature of the inverse of the terms. So, we will further impose the condition that the inver/ se metric follows

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

[3]The wave operator will have respectively the signature of the defined metric. Here, it is -+++.

[4]T-R refers to the "Trace-Reversed" perturbation.

Here, the symmetric tensor $\Theta^{\mu\nu}$ is the tensor that identifies the polarisation effect. Also, the propagation vector ψ_λ shows the direction and the frequency of the wave propagation. The Hilbert gauge condition allows us to define the above two terms that represent the intrinsic nature of the wave in amplitude and frequency to be *orthogonal* in that their direct product is zero. Therefore, the polarisation tensor $\Theta^{\mu\nu}$ really has 6 independent terms.

One really important point here is that the gravitational field tensor can be written down as a TT[5] gauge, depicted as $\widetilde{h}^{TT}_{\mu\nu}$ which has two significant results: firstly, it is *trace-less*, which means that the sum of all the diagonal elements in the matrix is zero. Secondly, the waves is transverse to the direction of propagation, and these added to the gauge freedom lists out the following relation (1.1.6)

$$\hat{h}^{TT}_{\mu\nu} = h^{TT}_{\mu\nu}$$

Further, this also means that there are only two instead of the reduced six components in the polarisation tensor. The resultant polarisation shows the effect of a propagating gravitational wave on a ring of particles. If we have a *plus polarised wave* then the resultant ring of particles will expand and contract in the form of a plus, like wise for a *cross polarised wave*. The polarisation tensor can be written in terms of unit polarisations in the form of the matrices. We can write out the polarisation tensor as: (1.1.7)

$$\Theta^{\mu\nu} = h_{plus}\omega^{\mu\nu}_{plus} + h_{cross}\omega^{\mu\nu}_{cross}$$

Here, the coefficients h_{plus} and h_{cross} are the only two non-zero elements in the matrix h .

Now, we will take a deeper consideration of the gravitational field tensor and the metric perturbation addition tensor from the linearized Einstein Field Equations, and then we will write out the relation to the physical events that produce gravitational waves.

We can write the linearized Einstein Field Equations as the set, (1.1.8)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

We will make the following expressions look better by setting $c = 1$. In the gauge setting we have used where we get degrees of freedom for the gauge under the infinitesimal harmonic transformations set, we can write out the Field Equations to be of the form (1.1.9)

$$\Box \hat{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

The gauge setting is such that as said above, we still have some freedom in the gauge, and the Riemann tensor can be written out. The Field Equations reduce to the form (1.1.10)

$$\Box \hat{h}_{\mu\nu} = 0$$

And with that, the Riemann tensor can be expanded to give (1.1.11)

$$R_{\alpha\beta\gamma\delta} = \frac{-1}{2}\ddot{h}^{TT}_{\alpha\gamma}$$

Quantum Field Theories (QFTs) in themselves suggest a boson for gravity, a *graviton*. In our linearized theory in gravity, we notice the perturbations are invariant under rotations of an angle of 180° , meaning that they are spin-2. Further, these waves are massless and propagate at the speed of light. In QFT, gravitons are designated as spin-2 particles without mass.

The significance of the perturbation-gravitational field tensor is in that it helps us to understand the detection of the gravitational waves, and therefore physical numeric values of the source. The separation between the test masses used in the detection changes if a gravitational wave passes through the detector, and the initial-final separations can be found out by writing out the TT gauge in the indices of the respective separations. Simply, the following equation encodes the separations: (1.1.12)

$$\delta d^\alpha = \frac{1}{2} h^{TT}_{\alpha\beta} d^\beta$$

This is very important, because it shows the change in the separation of the test masses.

We will now discuss about approximations, and how we can understand the dynamics of gravitational waves mathematically, and then eventually we will consider wave detection and detector optimisations.

1.2 GRAVITATIONAL WAVE APPROXIMATIONS FROM SOURCE

The nature of the wave depends on the nature of the source. The set of all those terms in the metric perturbation that are quadratic are considered in the Field Equations by writing out the Field Equations

[5] TT refers to Transverse-Traceless. Traceless refers to that the individual sums of the matrix diagonal elements is zero.

in terms of the perturbation. We have already derived this result in the equation (1.1.9). Following this result, we can write down the *Post-Newtonian approximation*[6] in terms of the quadratic terms as (1.2.1)

$$\Box h^{\mu\nu} = -16\pi G \Sigma^{\mu\nu}$$

Here, the term $\Sigma^{\mu\nu}$ contains all the respective quadratic terms embedded. The complete expansion results in an important result.

The emission off gravitational waves the motion of matter coalescing changes. This causes a change in the orbital period of the system. The 1974 Hulse and Taylor observation of the pulsar system PSR1913+16 indirectly showed the properties of gravitational waves. Since the system has a pulsar, it aided the observation as a "clock" in that it helped to understand the *draining* of the system's period, thereby confirming that the effect of gravitational waves in a merger as predicted by General Relativity.

A binary system has two bodies, and they revolve around the gravitational centre of the system. In many cases, there have been observations indicating that one of the composite stars is heavy enough to form a gravitational singularity. In some cases, *both* the bodies are black holes, in which case they can inspiral towards each other and form a final settled black hole. The result is a black hole described by the Kerr solution. This describes a black hole system that exhibits both mass and angular momentum, two of the three numbers that can identify black holes by the No Hair theorem. The other characteristic property of black holes is an electric charge, which is described by the Reissner-Nordstrom solution. It is easy to know whether a system comprises of black hole mergers. In such cases, the final separation between the bodies is in terms of the Schwarzschild radius, and this happens in the merger at a time they are still some distance described by the Schwarzschild Radius away. This primary characteristic was used to show that the source GW150914 was a black hole merger. The final state comprised of a black hole that settled only a few times the Schwarzschild radius. This meant that the binary system which was the origin of the gravitational waves *must* have been a black hole merger.

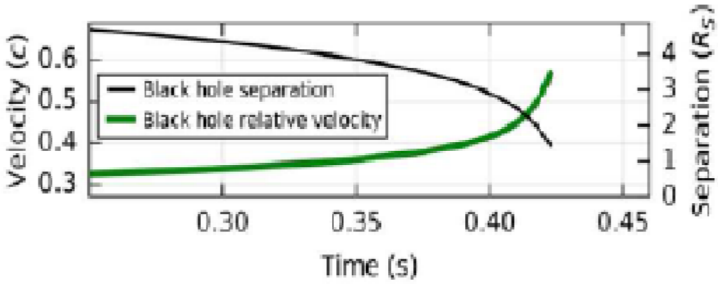

Fig-1: The merging of the black holes as depicted by a velocity-time-separation graph. It is easy to note that the velocity-separation intercept is the merger stage, and since the separation at that point is between two and three times the Schwarzschild Radius, this is a clear depiction of a black hole merger.

2. GRAVITATIONAL WAVE DETECTOR ARRANGEMENT AND INSTRUMENTATION

2.1 DETECTOR ARRANGEMENT

The primary foundation of gravitational wave detection is interference patterns. The base of detection lies in the effect of a gravitational wave passing through the detector. The detector uses a light ray, splitting and recombining them, and if a wave passes through the detector, the "test masses" change slightly in the length. By recombining, the waves are slightly perturbed, which means there will be an *interference* pattern observed. The expected change in the pattern is very small, less than 10^{-5} times the width of an atom, which means that the detector in itself is a very sensitive instrument. Any "noises" from the photon-shot of the light source, environmental or seismic events, or the detector error margins itself. The initial Laser Interferometry Gravitational wave Observatory (iLIGO) was built with an aim to provide results that could filter background noise from the true signal it pickeed up . Later, it was shut down, and the advanced Laser Interferometry Gravitational wave Observatory was built in its place.

[6] Einstein, Infeld, Hoffman. The Post-Newtonian approximation equations of motion have to do with weak gravitational fields where the individual motion of the bodies is comparatively slow as to the speed of light.

The detector is based on the Interferometer design by the Michelson-Morley experiment, and is hence known as the Michelson-Morley Interferometer. If a gravitational wave passes through the detector, then an interference pattern will be observed. This follows from the orthogonal property of gravitational waves. This is the reason the detector "arms" are at right angles to each other; if a gravitational wave passes, then one of the "arm" stretches, while the other gets shorter.

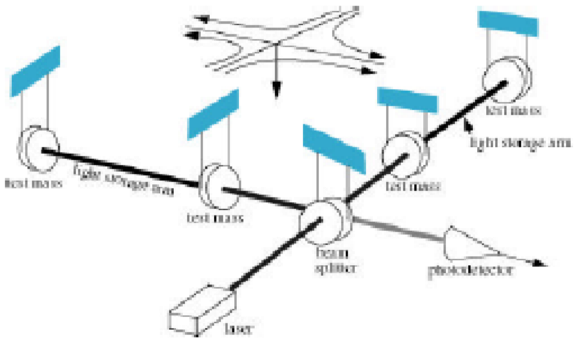

Fig-2: A schematic representation of the working of the Michelson-Morley Interferometers used at LIGO Hanford and Livingston. A source shines light on a beam splitter, which splits the beam into the "arms" of the detector. The test masses are suspended freely as a pendulum. The recombination usually results in no pattern, as the respective waves are out of phase. If a gravitational wave passes through the detector, there is an interference pattern observed.

The expected waveforms are many. For instance the *Chirp* waveform, found in black hole binaries and the *Impulse* waveforms, found in those of black hole mergers have been simulated and have been calculated up to a large precision by the mathematical modelling.

2.2 DETECTOR OPTIMSATION

The detector is a very sensitive instrument. It can pick up even minor disturbances in the surroundings of the detector. Therefore, there is a problem of "false detection noises" that can arise. These noises could be from the detector itself, it could be from the geological activities around the detector, or even the noise of the light beam. Therefore, there is a fundamental need to optimise the detector from false alarms. In general, there is a problem with the following:

1) Seismic wave systems
2) Detector surrounding noises
3) Test mass "ringing"
4) Mirror "ringing"

Seismic waves are caused by geological activities, and these pose a very big problem to detection. Likewise, any environmental disturbances can also be picked up by the detector, making it difficult for any merger or inspiral noise to be detected. Further, the test mass which is suspended can "ring" , which makes it a serious difficulty to filter. Similarly, any vibrations in the instrument will strain the detector from filtering an astronomical event. Therefore, it is important to be able to remove any additional noises from being detected by the instrument.

The Livingston and Hanford LIGO detectors are built in seismically isolated areas, thereby preventing the problems that arise with the mixing of these sounds. The detector is further built so that it responds actively to a gravitational wave frequency, yet not so free as to possibly vibrate. Therefore, if a frequency of more than a certain limit is achieved, the detector is then "free" , and therefore can detect it. The test masses are "locked" , which means that the test masses have to be held in position to allow the instrument to pick up a *true* astronomical event, and to prevent noises arising from any possible vibrations of the test masses. Further, it is required to identify the minimum optimised frequency that relates to a *true* cosmological event.

The advanced LIGO detector was optimised to identify lesser possible background noises than the initial LIGO detector. The optimisations were in the following points:

1) Sensitivity increment up to 5 times iLIGO
2) Seismic shock stacking to prevent any seismic noises from being picked up
3) Heavier mirrors to filter thermal noises
4) Prevention of quantum noises by increasing the power of the laser beam

This set of upgrades meant that the detectors were sensitive enough to pick up even the most sensitive noises, and also optimised to filter out any background noises coming from the detector or the surroundings. This was done by increasing the test mass diameter and mass. The iLIGO had a test mass diameter of ≈ 25 cm and mass of ≈ 11 kgs. The aLIGO had been optimised by increasing the test mass diameter to ≈ 34 cm and mass of ≈ 40 kgs. This meant that since the test masses were comparatively larger and massive, they could prevent the photon impact vibrations, thereby filtering out vibrational noises from the test masses. Further, the iLIGO test masses were suspended using metal wires, which meant that vibrations in them could add up to the interference, which meant that the detector was again picking up additional unwanted noises. The aLIGO detector was upgraded to be suspended using glass fibres, which essentially dampened unwanted vibrations in the suspension. Further, the iLIGO was suspended singularly as a pendulum, whereas the aLIGO was suspended in the form of a quadruple pendulum, and this decreases any vibrations from any part of the suspension.

These adjustments allowed aLIGO to detect gravitational waves within days of the first test run. Labelled GW150914, it was detected on the 14$^{\text{th}}$ of September, 2015. It was found out to have originated from a black hole merger, and we will be considering the data analysis and the exact numeric values that have been observed in the event GW150914 in the next section

3. AN EXAMPLE: GW150914 DETECTION

GW150914 was the first astronomical event to be detected using the LIGO detectors, and this happened within days of the first test run. The detection data has been shown to be of a waveform which originated from the merging of two black holes. In this section, we will discuss the primary data from the detection of gravitational waves from the source.

The detection from the event that took place 1.3 billion light years away was filtered using a process called *match-filtering*, and by reconstructing the wave templates. The detection shows that the waves had originated from the inspiral of two black holes.

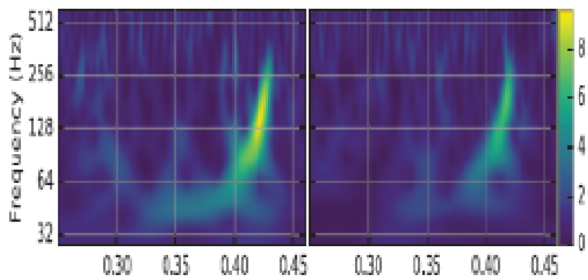

Fig-3: The observation of the gravitational waves depended on filtering the templates to find the inbuilt gravitational waves. The left side shows the Hanford detection and the right side shows the Livingston LIGO detections. The detection is found to have originated from a *real* astronomical event by the coincidental detection by *both* the detectors.

At 09:50:45 UTC, there was observed an increase in the frequency-time plotting in both the detectors as shown in figure-3. The match-filtering from numerical-relativity construction was used to split the gravitational strain, thereby calculating the exact waveform residue. The comparison between the Hanford and the Livingston observed spikes show that the detection is different than any unfiltered noises by up to 99% , thereby confirming up to a large precision that this was indeed a *real* event. The error range was very less, and the detected bound was greater than 5σ , meaning further that this couldn't be a false alarm detected.

The detected waveform at the Hanford LIGO detector is depicted in figure-4a, and the comparison to the Livingston LIGO detector is shown in figure-4b.

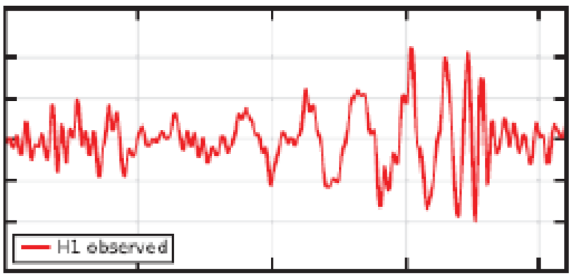

Fig-4a: The gravitational wave detection GW150914 by the Hanford LIGO detector.

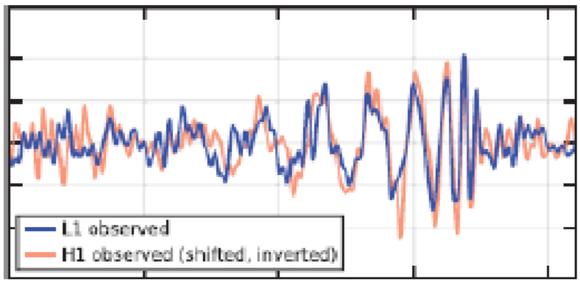

Fig-4b: The gravitational wave detection GW150914 by the Livingston LIGO detector is depicted in blue while the waveform detected by the Hanford detector is depicted in red to provide a comparison.

The numerical filtering has been taken in to consider the gravitational strain (in the order of scale

10^{-21}). By comparing to expected binary forms, the residual waveform then is found out to be as in figure-5.

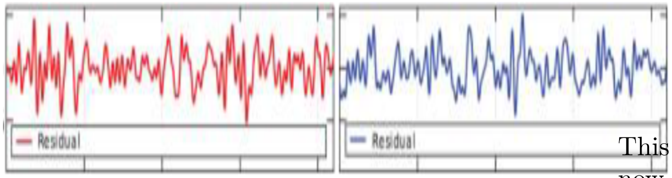

Fig-5: The residual waveform of GW150914 as found out by match-filtering and comparing to compact binary templates.

This detection has been found to have originated from a black hole inspiral. This was found by taking the numerical relativity approximation and comparing it to the observed strain, and it was found to match the construction of a black hole merger (see figure-6a and 6b).

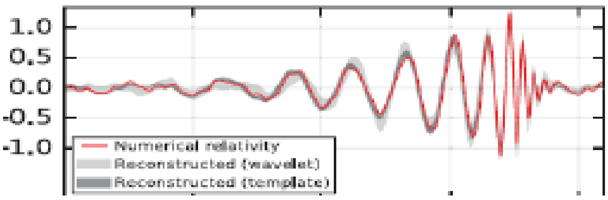

Fig-6a: Comparing the numerical construction of a black hole merger to the event GW150914 as observed from the Hanford LIGO detector. The observed and the numerical approximations match. The eventual building-up of the strain-time graph is the inspiral stage, and the final "burst" is the merging of two black holes.

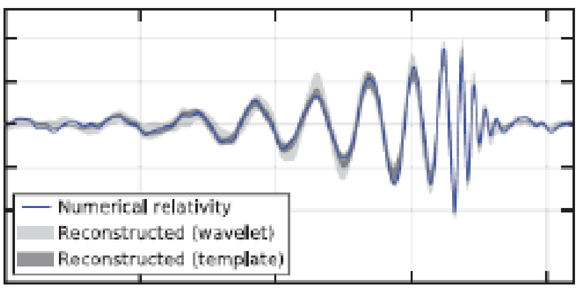

Fig-6b: Comparing the numerical construction of a black hole merger to the event GW150914 as observed from the Livingston LIGO detector.

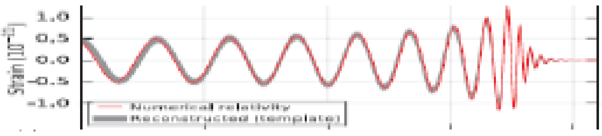

Fig-7: The numerical simulation of a black hole inspiral and merger event.

This points towards a black hole inspiral. We will now look further into the data this points at. We will now talk about the nature of the event.

The calculation of the chirp mass from the data from the detectors help to find out the nature of the event producing the detected gravitational waves. From the detection, there is an observed chirp mass of approximately $30M_\odot$. In order to reach the frequency as observed, the objects should only be a few times the Schwarzschild radius, and very compact. Although neutron stars are also candidates for the event GW150914, these have been ruled out since they do not have enough mass to reach the detected approximate value of greater than $62 \sim 70$ times the mass of the sun, meaning that black holes were now the only possible candidates. From chirp mass calculations (3.1),

$$M_{Chirp} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$$

the individual masses can be approximated to be around 35 and 29 times the mass of the sun. The black holes finally merge to form a stationary black hole which is described by the Kerr solution. The final spin angular momentum of this can be estimated to be ≈ 0.67

GW150914 was the first gravitational wave detection by the LIGO detectors, including a detection GW151226[7], the same year, which comprised of masses of around 14 and 7.5 times the mass of the sun[8]. Since then, several such events have been detected. LIGO will be spreading its field of vision of the sky by installing detectors and counterparts[9] around the Earth. These are expected to open the windows to explore the cosmos and the fine mathematical ripples that general relativity predicted as far as in the early universe. 20^{th} century.

CONCLUSION

In this paper, we have discussed the various points that are required to construct the mathematics of

[7]This event too was comprised of a black hole inspiral, but with lighter individual black hole masses.

[8]This event further gave rise to a final black hole system of mass around 20 times the mass of the sun. The final spin angular momentum was estimated to be .

[9]LIGO India is expected to be online in the next few years. KAGRA, the Japanese gravitational wave detector and the VIRGO detector in Italy are optimistic to be fully operational and working to detect further astronomical events that might generate gravitational waves.

gravitational waves. We have taken a mathematical overview of the structure of gravitational waves. We have first constructed the linearity of the Einstein Field Equations, from which we wrote down the Christoffel symbols and the Riemann tensor. We have then considered the metric perturbation tensor, which allows us to define the gravitational field tensor, as discussed using the trace of the perturbation tensor, $h_{\mu\nu}$. We have further discussed the effect of propagation of a gravitational wave on a ring of particles, under the Polarisation tensor. The Polarisation tensor splits into two terms, whose polarisation is either plus or cross, depending on the effect of the wave propagating. We have then discussed about gravitational field approximations from the source, and have highlighted the topic of Binary Black hole Systems (BBHS), and discussed the properties of such Binary systems. We have then discussed Detector Arrangement and Detector Optimisation, and then we have discussed about the first event detected, $GW150914$, which was detected using match-filtering to be a BBHS merger. We have discussed the working of the analysis of this event in particular.

Our primary focus has been on an overview of the concept of gravitational waves. We have discussed the observation $GW150914$. We have also discussed the advances so far in gravitational waves, where we talked about the optimisation of Laser-based Interferometer detectors

A final note would be that General Relativity is one of the most important discoveries by man. A poem simple yet complex, General Relativity is being known more and more, and the mathematical beauty of it shall be explored further.

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