Geometrical appearance of circumference as statistical consequence

Date: 2017-01-05

Wolfgang Orthuber
University Kiel, UKSH
orthuber@kfo-zmk.uni-kiel.de

Abstract— Because identical fermions (elementary particles) have (except spacetime coordinates) exactly the same features everywhere, these are (per proper time) a multiple mapping of the same. This mapping also leads to the geometrical appearance (of spacetime) and it provides a set of possibilities which can be selected (like "phase space"). Selection of possibilities means information. New selection of possibilities means decision resp. creation of information. This paper should motivate to a more consequent information theoretical approach (not only in quantum mechanics but) also towards spacetime geometry. It is a short supplement to previously published material, where it was shown that proper time is proportional to the sum of return probabilities of a Bernoulli Random Walk. The probabilities at every point in such a walk result from "OR" operation of incoming paths. The probability of a "AND" operation at a certain point can be interpreted as meeting probability of two simultaneous and independent Bernoulli Random Walks. If no direction is preferred (p=1/2), after n steps this meeting probability (of two simultaneous symmetric Bernoulli Random Walks resp. BRWs) in the common starting point goes for large n to 1/(2πn), which is the inverse of the circumference of a circle with radius n. So if a BRW pair denotes two commonly starting simultaneous independent BRWs (each with p=1/2), after n steps (in case of large n) in the average 1 of 2πn BRW pairs meet again in its original starting point.
Likewise due to the limited speed of light our knowledge of surrounding is the more delayed, the greater the distance n is. Therefore there are the more (geometric) possibilities of return (2πn possibilities for multiples of the same fermion on a circle with radius n), the greater the distance (the radius) n is. This shows a basic example for a connection between statistical results and geometrical appearance.

Keywords — Bernoulli Random Walk, BRW, quantum physics, statistics, circumference, circumferential length, circle, geometrical appearance, spacetime geometry

---

INTRODUCTION

Due to quantum physical results it is reasonable to assume that geometry of spacetime has a discrete (and statistical) origin. A basal geometric feature is the nontrivial proportionality factor 2π between radius and circumference of a circle. Here we show a short statistical approach to this proportionality factor.

APPROACH

A Bernoulli Random Walk is generated by a sequence of independent trials or "steps" [Fe] [Sp], each one of which can have two results, e.g. "positive" (with probability p) or "negative" (with probability 1 - p). We can interpret it as model for the movement of a particle in a one-dimensional lattice of equidistant points or "states" which are indexed by an integer coordinate k. With every trial the particle makes a step from point k to point k + 1 with given probability p ("positive direction") or a step from point k to point k - 1 with probability 1 - p ("negative direction"). As in [O2] for $n \in \{1,2,3,\ldots\}$ we denote by Q0P(n, k, p) the probability, that the particle is at point k after the n-th step and by Q0P(0, k, p) this probability before the first step. We assume start of movement at k = 0, so Q0P(0, 0, p) = 1 and Q0P(0, k, p) = 0 for $k \neq 0$ and furthermore

$$Q0P(n + 1, k, p) = p\, Q0P(n, k - 1, p) + (1 - p)\, Q0P(n, k + 1, p) \qquad (1)$$

When making n trials, point k is only within reach, if n - k and n + k are non-negative even numbers. We will presuppose this subsequently. There are exactly n!/(((n+k)/2)! ((n-k)/2)!) paths with (n+k)/2 steps in positive and (n-k)/2 steps in negative direction, which lead into point k after the n-the step. They respectively have the probability $(1-p)^{(n-k)/2}\, p^{(n+k)/2}$. So the chaining of these Bernoulli trials results into the binomial distribution

$$Q0P(n,k,p) := \frac{(1-p)^{(n-k)/2}\, p^{(n+k)/2}\; n!}{\left(\frac{n-k}{2}\right)!\left(\frac{n+k}{2}\right)!} \qquad (2)$$

Subsequently assume assume p=1/2 and define $Q0(n,k) := Q0P(n,k,\frac{1}{2}) = \dfrac{n!}{\left(\frac{n-k}{2}\right)!\left(\frac{n+k}{2}\right)!\; 2^n}$ (3)

By BRW we denote a Bernoulli Random Walk with p=1/2.

Q0(n,k) represents probabilities in case of p=1/2. In the symmetry center we get $Q0(n,0) = \dfrac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!\; 2^n}$ (4)

Fig. 1 shows the Q0(n,k) which represent the probabilities of a BRW (Bernoulli random walk with p=1/2).

n ↓ \ k->	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	
0										1										*1/1
1									1		1									*1/2
2								1		2		1								*1/4
3							1		3		3		1							*1/8
4						1		4		6		4		1						*1/16
5					1		5		10		10		5		1					*1/32
6				1		6		15		20		15		6		1				*1/64
7			1		7		21		35		35		21		7		1			*1/128
8		1		8		28		56		70		56		28		8		1		*1/256
9	1		9		36		84		126		126		84		36		9		1	*1/512
...																				

Fig. 1 Probabilities of a BRW (symmetric Bernoulli random walk with probabilities p=1-p=1/2 for both sides). The probabilities in the central column k=0 are underlined. Conservation laws suggest a natural privilege of these central states. The probabilities of the inflowing paths are in the columns with k=-1 and k=1.

The probabilities of the 2 (left and right) paths into the center are

$$Q0(n-1,1) = Q0(n-1,-1) = \frac{(n-1)!}{\left(\frac{n-2}{2}\right)!\left(\frac{n}{2}\right)!\ 2^{(n-1)}} = Q0(n,0)\frac{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!\ 2}{\left(\frac{n-2}{2}\right)!\left(\frac{n}{2}\right)!\ n} = Q0(n,0) \qquad (5)$$

It is $Q0(n,0) = \frac{1}{2}Q0(n-1,1) + \frac{1}{2}Q0(n+1,1)$ because Q0(n,k) is an OR-operation of both incoming paths (from Q0(n-1,k+1) plus from Q0(n-1,k-1)). This defines a BRW.

Suppose that two symmetric BRWs (BRW1 and BRW2) start simultaneously and are stepping simultaneously.

First we assume that the sum of all k is constant (symmetry around k=0, conservation law). In this case we know: If k increases in BRW1, then k decreases in BRW2, and reverse. If at start k=0, there is complete symmetry. We can assume that one of both BRWs moves freely and the other totally depends on it. If one BRW arrives at k=0, then also the other. So the meeting probability is the return probability of a symmetric BRW:

$$Q0(n,0) = Q0(n-1,-1)/2 + Q0(n-1,1)/2 \qquad (6)$$

Now suppose that two BRWs again start in k=0 and step simultaneously, but step directions (k+1 or k-1) are done independently. Let Q0AND(n,k) denote the meeting probability of two such BRWs with independent step directions. In this case the probability that one arrives after n steps at k=0 is Q0(n-1,-1)/2, and that the other arrives at k=0 is Q0(n-1,1)/2. Because steps are done independently, the probability Q0AND(n,0) that both meet in k is due to (5):

$$Q0AND(n,0) = \frac{Q0(n-1,-1)}{2}\frac{Q0(n-1,1)}{2} = \left(\frac{Q0(n,0)}{2}\right)^2 \qquad (7)$$

Equivalently we can suppose to do the split into two halves directly in the start, so that every half is an independent BRW with half probability. In point (n,k) it is Q0(n,k)/2 which again leads to the combined probability (7).

Using the Stirling formula $\lim_{n\to\infty} n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ we get for large n

$$Q0(n,0) \approx \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\left(\sqrt{\pi n}\left(\frac{n}{2e}\right)^{n/2}\right)^2 2^n} = \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\pi n\left(\frac{n}{2e}\right)^n 2^n} = \sqrt{\frac{2}{\pi n}} \qquad (8)$$

and for large n so from (7)

$$Q0AND(n,0) \approx \left(\sqrt{\frac{1}{2\pi n}}\right)^2 = \frac{1}{2\pi n} \qquad (9)$$

From this follows (for BRWs with no preferred direction and large n)

Formulation 1:
The meeting probability of two commonly starting simultaneous independent BRWs after n steps in their common starting point goes for large n to 1/(2πn), which is the inverse of the circumference of a circle with radius n (or the probability to meet a segment of length 1 on a circle with radius n).

More demonstrative may be the viewpoint after "renormalization". Implicitly we make within every perception a renormalization. The "probability" of an altogether very improbable perception is renormalizated to 1. According to the following formulation 2 the factor for such renormalization after n steps can be just 2πn:
Formulation 2:
If a *BRW pair* denotes two commonly starting simultaneous independent BRWs, after n steps (in case of large n) in the average 1 of 2πn BRW pairs meet again in its original starting point.

This is interesting because it shows a relatively simple connection between statistics and geometry. If both BRWs start simultaneously and the sum of k is conserved (symmetry), the return probability (6) is also a meeting probability ("OR" operation). If, however, the BRWs start (later) simultaneously and decide independently ("AND" operation, (7)), the probability that they meet after n steps in the starting point k=0 is the geometrical probability Q0AND(n,0) which is the inverse of the circumference of a circle with radius n.

The following supplementary chapters are added to show possible connections to current models.

INTERPRETATION, THOUGHTS FOR FURTHER THOUGHTS

At first the above approach seems to be only 2D (two-dimensional) because circumference is contained in a 2D plane. But this fits to propagation of electromagnetic fields. With (3) p=1/2 and according to [O2] this is connected with the propagation speed v=c (speed of light). So we can assume electromagnetic interaction. At this for example the direction of inducing resp. induced electric currents and changes in electric fields are proportional to the circulating magnetic fields. The 2D plane of a circulating (magnetic or electric) field is shown (resp. determined) by the direction of the inducing resp. induced (electric or magnetic) field.
The 3D propagation of information results after more steps.

Due to the limited speed of light our knowledge of surrounding is the more delayed, the greater the distance n is. Therefore there are the more (geometric) possibilities of return (2πn possibilities for multiples of the same fermion on a circle with radius n), the greater the distance (the radius) n is.
So the above approach also shows first steps to answers of the following questions:

- Why are there conservation laws?
  - Because completed perception at last is only possible inside a symmetry center (k=0, see Fig. 1).
- Why is v=c (Why is the maximal information speed constant and finite)?
  - Because a well defined delay (at least n>=2 in Fig. 1) is necessary for statistical development of geometry, i.e. for freedom of geometrical coordinates in surrounding.
- Why do the same fermions have exactly the same features everywhere?
  - Because during statistical development of geometry multiple possibilities (geometrical coordinates) lead (back) to the same elementary constellation.
- What is the information theoretic origin of the proportionality factor π in geometric formulas?
  - see (9). Due to limits (8) and (9) the occurrence of π in geometric formulas (e.g. the proportionality factor 2π between radial distance and circumference) indicates a combination (concatenation or "AND" operation) of two statistics (BRWs).

Two past[1] BRWs compared to what? One step forward is more probable than a series of 2 steps back - this could define an order. Interpretation of experimental results concerning definition of time direction?

QUESTIONS FOR CONTINUATION

As already mentioned above, for description of 3D propagation of information more steps are necessary. How can we extend the (information theoretical) approach to 3 dimensions which represent statistically nearly uncorrelated quantities?

Connected questions:

- Information theoretical interpretation of basal (discrete) Maxwell Equations? We could study their development using varying conditions.
- Connection to basal (discrete) Schroedinger Equation?
  In connection with the Schroedinger Equation it is noteworthy that
  (Q0(n,k-2)-Q0(n,k)) - (Q0(n,k)-Q0(n,k+2)) = 4(Q0(n+2,k) - Q0(n,k))
  where the left side can be interpreted as discrete 2. derivation along location and the right side can be interpreted as discrete derivation along time.

---

[1] Geometry shows past (due to the limited information speed), so statistics which lead to geometry are past.

- Can the simplified low energy model of atomic shell (which starts in 2D) help as connection?

Constructive comment to current cosmological models

We should recall that a direct experimental evaluation of cosmological models is not possible. We cannot make experiments under conditions at very past time (e.g. with past physical constants). Therefore cosmological models are extrapolations. Current cosmological models ("Big Bang") extrapolate and start geometrically - despite the experimentally proven limits of geometrical models. Compared to this an approach with geometry as statistical consequence leads to completely different start conditions[2] and conclusions. We recommend to investigate these in more detail. Plausible would be to use from the beginning an information theoretical approach which develops into increasing complexity resp. branching depth. We can ask for the initial (most simple) situation of "information".

We know that information means selection from a set (of possibilities). A selection from a set with 0 elements is not possible. A selection from a set with 1 element (without alternative) provides no (new) information. So the most fundamental initial new information must describe selection of one element from a set with 2 elements.

The most fundamental set with this nature results from distinction between "past" and "presence". It seems that from this results order of time and secondarily order of other dimensions. A graph theoretic approach can provide deeper insight into multiple steps.

So it is recommendable to look in more detail and consequently for discrete definition of (local and global) time and to develop from this a contradiction free (information theoretical) interpretation of macroscopic geometrical appearance as statistical result.

Can we estimate maximal n?

How large may be n since start of our observable universe? In this chapter we try a rough guesswork:

Let age = $4.3*10^{17}$s (rough age of observable universe) and c= $3*10^{8}$ m/s (speed of light)

If we assume r = 10^{-15} m as (rough) diameter of a fermion (nucleon) and use this as minimal stepping size, we get c/r = $3*10^{23}$ steps per second and z:= age*c/r= $1.29*10^{41}$steps since start of the universe. Due to above Formulation 2 we assume that from n=1 to z with every step $2\pi n$ new possibilities are generated, then for the total sum of possibilities we get

$$\sum_{n=1}^{z} 2\pi n = 2\pi \frac{1}{2} z(z+1) \approx 5.2*10^{82}$$

This seems not so far away from the currently estimated count of nucleons in the observable universe.

Conclusion

An information theoretical approach which develops into increasing complexity resp. branching depth (with geometry as secondary statistical consequence) is more plausible than a primarily geometrical model (like "Big Bang"). It seems that geometric (macroscopic) physical measurements result from differentiation, superposition and concatenation of (meanwhile partially very large, periodically in a symmetry center synchronized) statistics.

References

[Ba] Barber M.N., Ninham B.W., Random and restricted walks, theory and applications, New York: Gordon & Breach, 1970.

[Fe] Feller, W. An introduction to probability theory and its applications, Vol. 1-2, New York: Wiley, 1957-1971.

[Fi] A. Fine, Theories of probabilities, an examination of foundations, New York: Academic Press, 1973.

[Ka] Kac, M., et al. Statistical independence in probability, analysis and number theory. Vol. 134. Washington, DC: Mathematical Association of America, 1959.

[Kh] Khrennikov, A., Volovich, Y., Discrete Time Leads to Quantum-Like Interference of Deterministic Particles, quant-ph/0203009.

[Kr] Krengel U., Einführung in die Wahrscheinlichkeitstheorie und Statistik, 3. erw. Au., Braunschweig: Vieweg, 1991.

[O1] Orthuber W., To the finite information content of the physically existing reality. arXiv preprint quant-ph/0108121.

[O2] Orthuber W., A discrete and finite approach to past proper time. arXiv preprint quant-ph/0207045.

[O3] Orthuber, W., A discrete and finite approach to past physical reality. International Journal of Mathematics and Mathematical Sciences, 2004(19), 1003-1023.

[O4] Orthuber W., The Recombination Principle: Mathematics of decision and perception (init. 2000, more comprehensive, with philosophical parts), http://www.orthuber.com

[Sp] Spitzer, F. Principles of random walk, 2nd ed., New York: Springer Verlag, 1976.

[2] For example concerning conditions at much earlier times: It is plausible that there was significant less branching depth which was connected with other physical constants. It would be interesting to look for possibilities to test the **hypothesis** that the quotient of comparable physical sizes (e.g. of electromagnetic and gravitational interaction of proton and electron) at much earlier times has been nearer to 1 or -1. Discrete sign conversion is possible.