Errata for my books, plus some afterthoughts

(Sterling K. Berberian, 15 September 2009)

The books, and their errata, are listed in reverse order of publication. The errata list all errors I have found in the first printing of the book in question; items decorated with an asterisk (*) received corrected reprintings:

Fundamentals of real analysis, Universitext, Springer-Verlag, New York, 1999. [MR **99i**: 28001]

A first course in real analysis, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1994.* [MR **95d**: 26001]

Linear algebra, Oxford University Press, Oxford/New York/Tokyo, 1992. [Zbl **0755**.15001]

Lectures in functional analysis and operator theory, Graduate Texts in Mathematics 15, Springer-Verlag, New York, 1974.* [MR **54** #5775]

Baer ∗-rings, Grundlehren der mathematische Wissenschaften, Band 195, Springer-Verlag, Berlin/Heidelberg/New York, 1972. [MR **55** #2983]

Notes on spectral theory, Van Nostrand Mathematical Studies No. 5, Van Nostrand, Princeton, N.J., 1966. [MR **32** #8170]

Measure and integration, Macmillan, New York, 1965* [MR **32** #1315]; reprinted by Chelsea, Bronx, N.Y., 1970.

Introduction to Hilbert space, University Texts in the Mathematical Sciences, Oxford University Press, New York, 1961* [MR **25** #1424]; reprinted by Chelsea, New York, 1976; reprinted by the American Mathematical Society, Providence, R.I., 1999; Spanish translation by Joaquin Sánchez Guillén, edited and revised by A. Plans Sanz de Bremond, Editorial Teide, Barcelona, 1970. [MR **51** #8799]

Notes on spectral theory received a "corrected 2nd édn." (2009), rekeyboarded by the author in TEX, with updated references; posted (as nst.pdf) on the University of Texas's web site for mathematical publications (www.ma.utexas.edu/mp_arc), as item 09-32 in the folder for 2009. {Google can find the site by searching for "mp_arc"; on the home page, searching for the keyword berberian will bring up all of the author's postings at this site.}

Fundamentals of real analysis, *Linear algebra*, and *Baer ∗-rings* have not received a corrected reprinting—the *raison d'être* of the present document.

Errata and comments for **Fundamentals of Real Analysis**

(S. K. Berberian, 19 July 2009)

I am indebted to Patrice Goyer for signalling the items preceded by an asterisk.

p. 24, ℓ. 5, 6. Pending the 'official' definition of the field $\mathbb{R}$ of real numbers in §1.8 (p. 32), the notations for intervals and the term 'real line' are to be interpreted in the sense of, for example, pp. 9 and 31 of *First course*.

p. 25, ℓ. 1,2. A better notation: replace y by x' (and reserve the letter y for elements of Y).

***p. 31,** ℓ. −14. At the end of line, for $(r_k - r_n)$ read $(r_k - r_n)'$.

p. 47. To footnote 2, add a reference to Th. 4.7 on p. 20 of the book of Hewitt and Stromberg (*op. cit.*), from which the proof given here is drawn.

p. 48, ℓ. −5. In fact (assuming the Axiom of Choice) the ordering on the quotient set is a simple ordering (by Th. 1.11.12, as noted in Th. 1.12.19 below).

p. 58, ℓ. 14. For (GC) read (CH).

p. 75, ℓ. 11. For $b < \infty$ read $b < +\infty$.

p. 88, ℓ. 14. For "invertals" read "intervals".

p. 89, ℓ. 10. A proof of the Heine-Borel theorem is given in *First course* (p. 76, Th. 4.5.4), and repeated below in Theorem 6.1.1 on p. 273.

p. 92, Exer. 3 of §2.1. Hint: Th. 2.2.1.

***p. 95,** ℓ. 10. Replace "second inequality" by "second equality".

p. 98, ℓ. 1, 2. Hint: Th. 3 on p. 27 of *Measure and integration* (cited henceforth as M&I).

***p. 101,** ℓ. 19. Read (vi) instead of (iv).

***p. 107,** ℓ. 28. In the 2-line display of Exer. 5, delete the expression $< \frac{2}{3}\lambda(\mathrm{U}_m)$ at the end of the second line; thus the display should read:

$$\begin{aligned}\lambda(\mathrm{U}_m - \mathrm{A}\cap\mathrm{B}) &\leqslant \lambda(\mathrm{U}_m - \mathrm{A}) + \lambda(\mathrm{U}_m - \mathrm{B}) = 2\lambda(\mathrm{U}_m) - 2\lambda(\mathrm{A}) \\ &< 3\lambda(\mathrm{A}) - 2\lambda(\mathrm{A}) = \lambda(\mathrm{A})\end{aligned}$$

{If $\mathrm{A}\cap\mathrm{B}$ were empty, the resulting inequality $\lambda(\mathrm{U}_m) < \lambda(\mathrm{A})$ would contradict the fact that $\mathrm{A}\subset\mathrm{U}_m$.}

p. 108, Exercise 8, (ii). Hint: 2.2.3, (v).

***p. 112**. In lines -3 and -1, replace $\mathrm{E}_1 \cap \mathrm{E}_2$ by E_2. Thus line -1 becomes

$$\mu(\mathrm{F}_2) = \mu(\mathrm{E}_1 - \mathrm{E}_2) + \mu(\mathrm{E}_2) \leqslant \mu(\mathrm{E}_1) + \mu(\mathrm{E}_2)\,;$$

p. 124, ℓ. 12. If $(x_n)_{n\geqslant 1}$ is a sequence and $(n_k)_{k\geqslant 1}$ is a sequence of positive integers such that $n_1 < n_2 < n_3 < \ldots$, then the sequence $x_{n_1}, x_{n_2}, x_{n_3}, \ldots$ is called a subsequence of (x_n) and is denoted (x_{n_k}).

***p. 150,** ℓ. 1. For (ii) read (iii).

p. 156, ℓ. -4. To the reference at the end of Remark 4.2.2, (v), add 2.2.3, (v).

***p. 179,** ℓ. 21. At the end of the line, for "of $\mathrm{X} \times \mathrm{Y}$" read "of X and Y, respectively".

p. 183. In 4.6.10, by 'open interval in $\mathbb{R}$' is meant an open interval with endpoints in $\mathbb{R}$, hence the intervals $\mathrm{I} \in \mathscr{I}$ are bounded (*First course*, *Examples* 1.3.2 on p. 9). {In contrast, $\mathbb{R}$ is the open interval $(-\infty, +\infty)$ in $\overline{\mathbb{R}}$, but it is not an interval in $\mathbb{R}$.}

***p. 185,** ℓ. 14. For "form" read "from".

Trivialities

p. 43,44. In 1.11.9, read *Proof* #1 and *Proof* #2 (i.e., suppress the periods after the word *Proof*; they were added by the copy-editor).

p. 108, ℓ. 15. Remove the brace "}" at the end of the line (it was inserted by the copy-editor); the closing brace for the Hint is at the end of line 25 (the copy-editor did not worry about its opening brace).

p. 129, ℓ. 7. For "transfomations" read "transformations".

Errata for **A first Course in Real Analysis**

(S. K. Berberian, 13 October 1997)

p. 4, Exer. 2. Instead of $\varphi : \mathrm{F} \to \mathrm{G}$, read $\varphi : \mathrm{F} \to \mathrm{F}$.

p. 87, ℓ. −3. Instead of $f(x)$ read $f(a)$.

p. 120, ℓ. 5–6. Instead of $x_n \neq x$ read $x_n \neq c$.

p. 174, ℓ. −6. Instead of $\mathrm{N}(\delta)$ read $\mathrm{N}(\sigma)$.

p. 187, Exer. 3, (i). Instead of "positive integers" read "nonnegative integers".

p. 193, Exer. 3. In the hint, replace the denominators by their square roots.

p. 204, ℓ. 6. Instead of $f(\mathrm{I})$ read $F(\mathrm{I})$.

p. 213, ℓ. 15. Instead of "negligence" read "negligibility".

p. 218, Exer. 3, (iv). The term "converse" should have been defined in Appendix A.1, as follows: Given a proposition "$P \Rightarrow Q$", its *converse* is defined to be the proposition "$Q \Rightarrow P$". In the exercise at hand, the converse of (iii) is the (false) proposition "F is strictly increasing $\Rightarrow f > 0$ a.e."

p. 228, ℓ. −11. Instead of $n' \in \mathbb{P}$ read $n' \in \mathrm{S}$.

Errata for **Linear Algebra**

(S. K. Berberian, 18 June 2008)

p. 53, ℓ. 19, (1) of proof.

"...by assumption, A is nonempty..." (suppress comma after the A)

p. 73, ℓ. -4.

"...subspaces..." (restore missing 's')

p. 77, Exer. 15.

"The following..." (restore missing 'n')

p. 136, Exer. 10, (ii).

Read: "(ii) $(\mathrm{A} \cup \mathrm{B})^{\perp} = \mathrm{A}^{\perp} \cap \mathrm{B}^{\perp} \subset (\mathrm{A} + \mathrm{B})^{\perp}$, with equality when $\mathrm{A} \cap \mathrm{B} \neq \varnothing$."

I owe this correction, and the clever observation about equality, to R. Ramabadran (Ahmedabad, India).

The inclusion can be proper if $\mathrm{A} \cap \mathrm{B} = \varnothing$. For example, in the Euclidean plane $\mathrm{E} = \mathbf{R}^2$, let $\mathrm{A} = \{e_1\}$, $\mathrm{B} = \{e_2\}$. Then $\mathrm{A}^{\perp} \cap \mathrm{B}^{\perp} = \{\theta\}$ but $\{e_1 + e_2\}^{\perp}$ is 1-dimensional.

Proof of $(\mathrm{A} + \mathrm{B})^{\perp} \subset \mathrm{A}^{\perp} \cap \mathrm{B}^{\perp}$ when $\mathrm{A} \cap \mathrm{B} \neq \varnothing$: Suppose $c \in \mathrm{A} \cap \mathrm{B}$. Let $x \in (\mathrm{A} + \mathrm{B})^{\perp}$. Then $x \perp c + c$, whence $x \perp c$. If $a \in \mathrm{A}$, write $a = (a + c) - c$; since $x \perp a + c$ and $x \perp c$, also $x \perp a$; thus $x \in \mathrm{A}^{\perp}$. Similarly $x \in \mathrm{B}^{\perp}$.

p. 272, ℓ. 20.

Read 11.1.8 (instead of 11.1.18).

p. 332, ℓ. 12.

"...one demonstrates..." (restore missing 'r')

p. 332, ℓ. 21.

"...demonstrated..." (restore missing 'r')

Errata and comments for

Lectures in functional analysis and operator theory

(S. K. Berberian, 23 February 1978)

Errata

p. 10, ℓ. 11. For e_0 read e_1.

p. 111, ℓ. 18. For "the convex generated by S" read "the convex cone (or the convex set) generated by S".

p. 111, ℓ. 19. For E read I.

p. 111, ℓ. 22. At the end of 27.14, add:
{Here $(y_i)_{i\in I}$ is any faithful indexing of the set S.}

p. 112, ℓ. 19. For $\dim_{\mathbb{R}} \geqslant 2$ read $\dim_{\mathbb{R}} \mathrm{E} \geqslant 2$.

p. 221, ℓ. −14. For $1+I$ read $1+M$.

p. 272, ℓ. 8. For "let us shown" read "let us show".

p. 317, ℓ. 16. For H read H.

Comments

p. 91. In Corollary (23.9), the hypothesis that E is separated is superfluous [23, Ch. I, 2$^{\mathrm{e}}$ ed., §2, No. 3, Cor. 2 of Th. 2].

p. 240. It follows from Th. 57.1 that T is a right-divisor of 0 in $\mathscr{L}(E)$ if and only if T' is a left-divisor of 0 in $\mathscr{L}(E')$. The chain of reasoning:

$$\begin{aligned} T \text{ is a right-divisor of } 0 &\Leftrightarrow T(E) \text{ is not dense in } E \\ &\Leftrightarrow T' \text{ is not injective} \\ &\Leftrightarrow T' \text{ is a left-divisor of } 0. \end{aligned}$$

p. 317, ℓ. 14, 15. The text is misleading: invariance of the Haar integral under $t \mapsto t^{-1}$ does not by itself assure that $\mathrm{L}^1(G)$ is commutative; but, for G abelian, invariance under $t \mapsto t^{-1}$ figures in the proof that $\mathrm{L}^1(G)$ is commutative.

"Smudges"

p. 178, ℓ. 4. Before the comma at the end of the line.

p. 234, ℓ. 9. On the symbol $f \circ g$.

p. 278, ℓ. 5. The period after $\|g \circ f\|_\tau$ is smudged.

Errata and comments for **Baer $*$-rings**

(S. K. Berberian, 27 August 2009)

ERRATA

p. 36, ℓ. 17. For "import" read "important".

p. 42, ℓ. 15. Read "For example:"

p. 100, ℓ. -21. In (i) of Exer. 3, read B instead of A.

p. 109, ℓ. -2. For GC read (P).

p. 119, ℓ. -20. In Exer. 14, read "involutory automorphism" in place of "automorphism".

p. 141, ℓ. 8. In (a) of Exer. 1, read "A *strict ideal* of A is. . ." (the initial capital letter should not be italicized).

p. 160, ℓ. -12. In (D3) of Prop. 1, for $D(h)$ read $D(h) = h$.

p. 242, ℓ. -3 to -1. Exer. 6A should have been placed in the next section, where the additional assumption $6°$ ensures that $\mathbf{C}$ has the property $\mathbf{x}^*\mathbf{x} \leqslant 1 \Rightarrow \mathbf{x} \in A$ (§54, Th. 1); granted this property, the proof given in [**6**, Th. 8] for finite AW*-algebras can be adapted to the present situation [the author, "Note on a theorem of Fuglede and Putnam", *Proc. Amer. Math. Soc.* **10** (1959), 175–182; the material *between* Th. 6 and Th. 8 is irrelevant here].

In §53, the exercise is an open question (the answer is not known to me in 2009); it should have been phrased as a question—"Do the relations . . . ?"—and it should have been labeled 6D instead of 6A.

p. 274, ℓ. 13. Assuming GC has been replaced by (P) in ℓ. -2 of p. 109, Theorem 3 should be added to Proposition 1 and Theorem 1 in the Hint. Note that (P) $\Rightarrow$ GC (referenced in the comments below).

p. 285, ℓ. -12. In the hint for "(e) implies (a)" of §56, Exer. 5, read $w \in A$ in place of $x \in A$.

p. 286, ℓ. -2. In the hint for §62, Exer. 9, in place of [§17, Exer. 17] read [§51, Exer. 17].

COMMENTS

p. 75, Ths. $3, 4, 5$. It suffices that A be a Rickart $*$-ring satisfying (SR) [S. Maeda, "On $*$-rings satisfying the square root axiom", *Proc. Amer.*

Math. Soc. **52** (1975), 188-190; MR 51#8158]; cf. Th. 12.13 on pp 56–57 of [B&B*][1]

p. 76, Exer. 16. See the preceding comment.

p. 76, Exer. 17. In a Rickart $*$-ring, the following conditions are equivalent: (a) every pair of projections in position p' can be exchanged by a symmetry; (b) for every pair of projections e and f, $u(ef)u = fe$ for a suitable symmetry u of the form $u = 2g-1$ with g a projection [S. Maeda, *op. cit.*].

p. 80, Prop. 7, its Cor. 2, and Th. 1. By a theorem of S. Maeda, every Baer $*$-ring satisfying the parallelogram law (P) also satisfies GC [S. Maeda and S.S. Holland, Jr., "Equivalence of projections in Baer $*$-rings", *J. Algebra* **39** (1976), 150–159; MR 53#8121]; cf. Cor. 13.10 on p. 61 of [B&B*]. Thus, in any proposition about a Baer $*$-ring that assumes (P) and GC, the assumption of GC is redundant (in particular, Maeda's theorem vaporizes Prop. 7). The examples noted below are not exhaustive.

p. 82, Exer. 5 and **p. 83,** Exer. 12. It suffices that A satisfy (SR), since (SR) $\Rightarrow$ (P) $\Rightarrow$ GC [Maeda and Holland, *op. cit.*]; cf. [B&B*], Th. 12.13 and Cor. 13.10.

p. 83, Exer. 17. The conditions (a), (b), (c) are equivalent in every Rickart $*$-ring; i.e., the assumption of orthogonal GC can be omitted [S. Maeda, letter to the author, October 8, 1974].

p. 83, Exer. 21. Yes; in fact, (P) $\Rightarrow$ GC in a Baer $*$-ring (referenced in the comment for p. 80).

p. 104, Th. 2. (P) $\Rightarrow$ GC.

p. 106, Prop. 5. Since (P) $\Rightarrow$ GC, the hypothesis (P) suffices.

p. 109, Exer. 12, (xi). Yes; in fact, the answer is yes for any Baer $*$-ring satisfying (SR) [Maeda and Holland, *op. cit.*].

p. 109, Exer. 15. (P) $\Rightarrow$ GC.

p. 110, Exer. 18. (P) $\Rightarrow$ GC.

p. 111, Remark. 4. (P) $\Rightarrow$ GC.

p. 115, Th. 3. (P) $\Rightarrow$ GC $\Rightarrow$ PC.

p. 117, Prop. 6. (P) $\Rightarrow$ GC.

p. 132, Exer. 11, (i) and (iii). The answers are "yes" for A a *finite* Rickart C*-algebra [D. Handelman, "Finite Rickart C*-algebras and their

[1] *Baer and Baer $*$-rings*, a 1992 update of *Baer $*$-rings*, posted (as baerings.pdf) on the University of Texas's web site for mathematical publications (www.ma.utexas.edu/mp_arc) as item 03-179 in the folder for 2003. Briefly, B&B*.

properties", *Studies in analysis*, pp. 171–196, Adv. in Math. Suppl. Stud., 4, Academic Press, 1979; MR 81a:46073].

p. 142, Exer. 10. Yes, if the algebra is finite (see the comment for p. 132, Exer. 11).

p. 144, ℓ. -13. In (viii) of Exer. 1, the notion of GC must be extended to accomodate 'formal projections' $1-e$ when A has no unity element.

p. 185, Th. 1. (P) $\Rightarrow$ GC.

p. 206, Th. 1. More generally, D. Handelman has shown that if A is a finite Rickart C*-algebra and M is a maximal ideal of A, then A/M is a finite AW*-factor (referenced in the comment for p. 132, Exer. 11).

p. 208, Exer. 3. More generally, every finite Rickart C*-algebra is strongly semisimple [D. Handelman, D. Higgs and J. Lawrence, "Directed abelian groups, countably continuous rings, and Rickart C*-algebras", *J. London. Math. Soc.* (2) **21** (1980), 193–202; MR 81g:46100].

p. 253, Exer. 2. For A a complex algebra with an involution (but no $\mathbf{C}$ in the picture), there is a far-reaching generalization by J. Wichmann [*Proc. Amer. Math. Soc.* **54** (1976), 237-240; MR 52#8947].

Errata for **Notes on Spectral Theory**

(S. K. Berberian, 2 September 2009)

Errata

p. 25, ℓ. 4. Formula (24) should read

$$\int f\,d\mu_{y,x} = \overline{\int \overline{f}\,d\mu_{x,y}}\,. \tag{24}$$

I.e., the integrand on the right side is the complex-conjugate $\overline{f}$ of f.

p. 28, ℓ. -9. The first line of the three-line display should have overbars over the second and third members, as follows:

$$(A^*x|y) = \overline{(Ay|x)} = \overline{\int f\,d\mu_{y,x}}$$

p. 88, Example *4. The example contains an incorrect statement; it should be revised as follows:

*4. Every regular weakly Borel PO-measure (Definition 15) is biregular [the author, "Sesquiregular measures", *Amer. Math. Monthly* **74** (1967), 986–990, Remark 5 on p. 989].

p. 115. The second sentence of the last paragraph should be revised as follows:

Granted the Naimark-Nagy dilation theory [10], there does exist a technique for representing an arbitrary operator A in the form

$$A = \int \lambda\,dF\,,$$

where F is a scalar multiple (by the scalar $\|A\|$) of a normalized compact PO-measure defined on the σ-algebra of Borel sets of the unit circle $|\lambda| = 1$ [see p. 181 of the author's expository article, "Naĭmark's moment theorem", *Michigan Math. J.* **13** (1966), 171–184].

Comment

The foregoing corrections are carried out in the "second edition", keyboarded in TEX and posted (as nst.pdf) on the University of Texas's web site (www.ma.utexas.edu/mp_arc) for mathematical publications; it is item 09-32 in the folder for 2009.

Errata and comments for **Measure and Integration**

(S. K. Berberian, 31 January 1965)

Errata

p. 63, ℓ. −10. The space between the words "in by" should be occupied by the symbol defined on p. 60, ℓ. −14 (the letter m surrounded by a 'tail', analogous to the 'at' symbol @ manufactured from the letter a).

p. 101, ℓ. −9. Read $M\mu(E_n) \leqslant \varepsilon$ instead of $M(E_n)\mu \leqslant \varepsilon$.

p. 288, ℓ. −2. Read $(x, y) \to (xy, y^{-1})$ instead of $(x, y) \to (y, yx^{-1})$.

Comments

p. 233. Questions in Exercises 3 and 5 of §70 have been answered in the negative by Alvin F. Martin [*Amer. Math. Monthly* **84** (1977) 554–555; MR **57**#12806]: there exists a locally compact Hausdorff space X such that every Baire measure on X is monogenic, but X admits (1) a Baire measure that is not completion regular, and (2) a Borel set that is not a Baire set. Example: X the 1-point compactification of a discrete space of cardinality $\aleph_1$ (the first uncountable cardinal).

Errata and comments for

Introduction to Hilbert Space

(S. K. Berberian, 10 August 1963)

ERRATA

p. 73, ℓ. −2. For $x \in \mathscr{S}$ read $x \in \mathscr{X}$.

p. 78, ℓ. −12. For $\int_a$ read $\int_a^t$.

p. 83, ℓ. 13. For "*of* T , *is*" read "*of* T *is*" (delete the comma).

p. 146, ℓ. 13. Read $\sum_{-\infty}^{\infty} \lambda_k y_{k+1}$ (the subscript on y was mangled in the first printing).

p. 158, ℓ. 12. For "**The** following" read "The following" (no boldface).

p. 179, ℓ. 17. For "R is self-adjoint" read "R is a self-adjoint".

p. 183, ℓ. 3. For " $\mu_n x_n \to y$," read " $\mu_n x_n \to y$;" (i.e., a semi-colon at the end of the line).

p. 184, ℓ. 6. For "*distinct proper values*" read "*distinct non-zero proper values*".

COMMENTS

The above misprints are corrected in subsequent printings. The following two items are noted in an *Addendum* on p. 202 of the Chelsea reprinting [Chelsea Publ. Co., New York, 1976]:

p. 188, ℓ. 5, 6. The question in item 10 on p. 188 has been answered in the negative by P. Enflo [*Acta. Math.* **130** (1973), 309–317]: a CC-operator in a Banach space may fail to be the uniform limit of finite-dimensional operators.

p. 188, ℓ. 7, 8. The answer to the question in item 11 on p. 188 is negative: that every hyponormal CC-operator is normal is a special case of a result of C. R. Putnam [*Proc. Amer. Math. Soc.* **7** (1956), 1026–1030, Cor. 3]. Further references are given in the *Addendum*.

p. 31, ℓ. 11. In the Chelsea printing, the word *resolvent* is replaced by *resultant*.