A resolution of Gordon ambiguity in nuclear current*

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Abstract

We investigate the electromagnetic vertex function of a nucleon in relativistic nuclear medium. In the corresponding Feynman diagram, the effect of mean-fields on the internal nucleon propagators connected to external ones offsets the difference between the familiar CC2 current and the so-called CC1 or CC3 current. It is therefore found that the CC2 current is physically reasonable. Consequently, the famous Gordon ambiguity of the nuclear current has been resolved.

In the electro (or photo) nuclear scatterings or reactions, nucleon current has the familiar Dirac plus Pauli form (called as CC2) determined phenomenologically in free space,

$$\Gamma^{\mu}_{CC2}\left(p_f, p_i\right) = F_1\left(q^2\right)\gamma^{\mu} + F_2\left(q^2\right)\left(i\,\sigma^{\mu\nu}q_{\nu}\right)/\left(2M\right), \tag{1}$$

where $p_{i(f)}$ is the initial and final momentum of a nucleon, $q = p_f - p_i$, M is free nucleon mass and $F_{1(2)}(q^2)$ is the Dirac (Pauli) form factor. Using the Gordon decomposition

$$\left(i\,\sigma^{\mu\nu}q_{\nu}\right)/\left(2M\right) = \gamma^{\mu} - \left(p_f + p_i\right)^{\mu}/\left(2M\right), \tag{2}$$

Eq. (1) is rewritten in the so-called CC1 form

$$\Gamma^{\mu}_{CC1}\left(p_f, p_i\right) = \left[F_1\left(q^2\right) + F_2\left(q^2\right)\right]\gamma^{\mu} - F_2\left(q^2\right)\left(p_f + p_i\right)^{\mu}/\left(2M\right), \tag{3}$$

or in the so-called CC3 form

$$\Gamma^{\mu}_{CC3}\left(p_f, p_i\right) = F_1\left(q^2\right)\left(p_f + p_i\right)^{\mu}/\left(2M\right) + \left[F_1\left(q^2\right) + F_2\left(q^2\right)\right]\left(i\,\sigma^{\mu\nu}q_{\nu}\right)/\left(2M\right). \tag{4}$$

However Eqs. (1), (3) and (4) are not equivalent for a nuclear nucleon. This famous Gordon ambiguity of nuclear current is the renewed interests in recent investigations of $(e, e'p)$ reaction [1-4] based on the Walecka $\sigma - \omega$ model [5] and the relativistic optical potential model.

*This paper is the revised version of CDS ext-2003-024. Although the physical contents are essentially the same, I have largely improved the context of the paper to clarify the physics.

Equation (2) is not an identity, while using the true identity

$$i\,\frac{\sigma^{\mu\nu}q_\nu}{2M}=\gamma^\mu-\frac{p_f^\mu+p_i^\mu}{2M}+\frac{\not{p}_f-M}{2M}\,\gamma^\mu+\gamma^\mu\,\frac{\not{p}_i-M}{2M}, \tag{5}$$

Eq. (3) is rewritten as

$$\Gamma^\mu_{CC1}\left(p_f,p_i\right)=\Gamma^\mu_{CC2}\left(p_f,p_i\right)-F_2\left(q^2\right)\left(\frac{\not{p}_f-M}{2M}\,\gamma^\mu+\gamma^\mu\,\frac{\not{p}_i-M}{2M}\right), \tag{6}$$

and Eq. (4) becomes

$$\Gamma^\mu_{CC3}\left(p_f,p_i\right)=\Gamma^\mu_{CC2}\left(p_f,p_i\right)+F_1\left(q^2\right)\left(\frac{\not{p}_f-M}{2M}\,\gamma^\mu+\gamma^\mu\,\frac{\not{p}_i-M}{2M}\right). \tag{7}$$

We can see that the difference between CC2 and CC1 or CC3 is just the second term of Eq. (6) or (7). It has no contributions to free positive-energy Dirac spinors (the $++$ coupling). However the Dirac spinor for a nucleon in nuclear medium described by the Walecka model contains negative-energy state [6] due to large scalar and vector mean-fields. Thus the second terms of Eqs. (6) and (7) contribute to the coupling between positive and negative-energy state (the $+-$ coupling).[1] It is an essential ingredient in the relativistic investigations of Refs. [1-4]. However, recent fully relativistic DWIA analyses of $(e,e'p)$ [7] and (γ,p) [8] reactions suggest that CC2 form is more appropriate than CC1 and CC3. Do they mean that the second terms of Eqs. (6) and (7) should be suppressed by the other medium effects?

So as to answer the question, we first resort to the renormalized Walecka model developed in Ref. [9]. There we derived the mean-field Lagrangian for symmetric nuclear matter as

$$\mathcal{L}_N=\bar{\psi}_{(0)}\left(\not{p}-M-\Sigma\left(p\right)\right)\psi_{(0)}, \tag{8}$$

where $\psi_{(0)}$ is the un-renormalized wave function. The self-energy $\Sigma\left(p\right)$ of nuclear nucleon includes quantum corrections.

$$\Sigma\left(p\right)=U\left(1+\xi\,\bar{U}\right)-\frac{1}{2}\,\xi\left[\bar{U}\left(\not{p}-M\right)+\left(\not{p}-M\right)\bar{U}\right], \tag{9}$$

where $\bar{U}=U/M$ and ξ is the isoscalar anomalous magnetic moment. (See Eqs. (28), (34) and (63) in [9].) The mean-field (or potential) U does not depend on the momentum and is composed of the scalar and vector parts:

[1] In nucleon knock-out reactions, initial bound nucleon, or the missing energy-momentum, is generally off (the mass) shell regardless of relativistic or non-relativistic model. Usually, an appropriate on-shell prescription is employed. As a result, there is an additional term [1] to Eq. (6). It however contributes to both the $++$ and $+-$ couplings and so is not our main interest. Hereafter it is neglected.

$$U = S + \gamma^0 V. \tag{10}$$

If the wave function is renormalized as

$$\psi_{(0)} = Z^{1/2} \psi_R, \tag{11}$$

using

$$Z^{-1} = 1 + \xi \, \bar{U}, \tag{12}$$

Eq. (9) becomes

$$\mathcal{L}_N = \bar{\psi}_R \left(\not{p} - M - U \right) \psi_R. \tag{13}$$

(See Eqs. (39) and (40) in [9].) It is noted that this renormalization is due to nuclear medium but not to the vacuum in free space.

Then we take into account electromagnetic interaction in Eq. (8). If the minimal substitution is simply adopted,

$$\mathcal{L}_N = \bar{\psi}_{(0)} \left(\not{p} - \not{A} - M - \Sigma(p) \right) \psi_{(0)}, \tag{14}$$

after the wave function renormalization we have

$$\mathcal{L}_N = \bar{\psi}_R \left(\not{p} - \tilde{\Gamma}^\mu A_\mu - M - U \right) \psi_R. \tag{15}$$

Because $\left| \xi \, \bar{U} \right| \ll 1$, the current becomes

$$\tilde{\Gamma}^\mu = Z^{1/2} \gamma^\mu Z^{1/2} \approx \left(1 - \frac{1}{2} \xi \, \bar{U} \right) \gamma^\mu \left(1 - \frac{1}{2} \xi \, \bar{U} \right) \approx \gamma^\mu - \frac{1}{2} \xi \left(\bar{U} \gamma^\mu + \gamma^\mu \bar{U} \right). \tag{16}$$

(See Eqs. (26) and (107) in [9].) This is just the isoscalar part of the CC1 current (6) at $q = 0$ using the Dirac equation from Eq. (13).

However Eq. (14) is not proper because the vertex should be also renormalized. For the purpose, we use the Ward identity

$$\Gamma^\mu = \left(\partial / \partial \, p_\mu \right) G(p)^{-1}, \tag{17}$$

where Γ^μ is the renormalized vertex and $G(p)$ is the propagator of a nucleon in the nuclear medium:

$$G(p)^{-1} = \not{p} - M - \Sigma(p). \tag{18}$$

Therefore

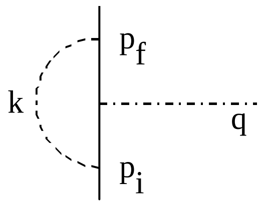
$$\Gamma^{\mu} = \gamma^{\mu} + \frac{1}{2}\,\xi\left(\bar{U}\,\gamma^{\mu} + \gamma^{\mu}\,\bar{U}\right) \approx \left(1 + \frac{1}{2}\,\xi\,\bar{U}\right)\gamma^{\mu}\left(1 + \frac{1}{2}\,\xi\,\bar{U}\right) \approx Z^{-1/2}\gamma^{\mu}Z^{-1/2}. \tag{19}$$

Replacing γ^{μ} in Eq. (16) by Γ^{μ},

$$\bar{\psi}_R\,Z^{1/2}\gamma^{\mu}Z^{1/2}\,\psi_R \rightarrow \bar{\psi}_R\,Z^{1/2}\Gamma^{\mu}Z^{1/2}\,\psi_R = \bar{\psi}_R\,\gamma^{\mu}\,\psi_R. \tag{20}$$

Consequently, for the renormalized wave function ψ_R, the original γ^{μ} vertex or the CC2 current is recovered. We have found that the difference between the CC1 and CC2 current is just the effect of wave function renormalization but it is canceled out by the effect of vertex renormalization due to the Ward identity.

So as to generalize the above consideration to the full form of CC1 current (6), we consider the first-order quantum correction, which is depicted by the next Feynman diagram, to the electromagnetic vertex function for a nucleon in symmetric nuclear matter:



Here the solid, dashed and dotted-dashed lines indicate nucleon ($G(p)$), meson ($D(k)$) and photon propagator respectively. This is expressed by

$$\Gamma^{\mu}\left(p_f, p_i\right) = \sum_a \int d^4k\,D_a\left(k\right)\Lambda_a\,G_f\left(p_f - k\right)\frac{1}{2}\left(1 + \tau_3\right)\gamma^{\mu}\,G_i\left(p_i - k\right)\Lambda_a, \tag{21}$$

where the index a indicates any of all necessary mesons and Λ_a is its vertex. The nucleon propagator satisfies Dyson equation [5,10]

$$G_{\,i(f)}\left(p\right) = G^{(0)}\left(p\right) + G^{(0)}\left(p\right)U_{i(f)}\,G_{\,i(f)}\left(p\right), \tag{22}$$

where $G^{(0)}(p)$ is the non-interacting Green's function. We assume that the mean field does not depend on the momentum and that the initial and final states are determined by respective one-body Hartree and optical potentials:

$$U_{i(f)} = S_{\,i(f)} + \gamma^0\,V_{i(f)}. \tag{23}$$

Substituting the iteration expansion of Eq. (22) and using identities [10],

$$\left(G^{(0)}\left(p\right)\right)^2 = -\frac{\partial}{\partial\,\not{p}}G^{(0)}\left(p\right), \tag{24}$$

$$G^{(0)}(p)\,\gamma^0 G^{(0)}(p) = -\frac{\partial}{\partial p_0} G^{(0)}(p)\,, \tag{25}$$

Eq. (21) is expanded as

$$\Gamma^\mu(p_f, p_i) = \sum_{n=0}^{\infty} \Gamma^\mu_{(n)}(p_f, p_i), \tag{26}$$

$$\Gamma^\mu_{(n)}(p_f, p_i) = \sum_a \int d^4k\, D_a(k)\, \Lambda_a \Upsilon^\mu_{(n)}(p_f, p_i, k)\, \Lambda_a, \tag{27}$$

where

$$\Upsilon^\mu_{(0)}(p_f, p_i, k) = G^{(0)}(p_f - k)\, \frac{1}{2}(1+\tau_3)\, \gamma^\mu G^{(0)}(p_i - k)\,, \tag{28}$$

and $\Upsilon^\mu_{(n)}$ for $n \geq 1$ is given by

$$\Upsilon^\mu_{(n)}(p_f, p_i, k) = \frac{1}{n!}\left[\,\Delta_i(p_i) + \Delta_f(p_f)\right]^n \Upsilon^\mu_{(0)}(p_f, p_i, k)\,, \tag{29}$$

with

$$\Delta_{i(f)}\left(p_{\,i(f)}\right) = -\left(S_{\,i(f)} \frac{\partial}{\partial \not{p}_{\,i(f)}} + V_{i(f)} \frac{\partial}{\partial p^0_{i(f)}}\right). \tag{30}$$

Consequently,

$$\Gamma^\mu(p_f, p_i) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\,\Delta_i(p_i) + \Delta_f(p_f)\right]^n \Gamma^\mu_{(0)}(p_f, p_i). \tag{31}$$

The terms of $n \geq 1$ in Eq. (31) explicitly extract the effects of mean-fields on the current. Although other medium corrections can be incorporated in $\Gamma^\mu_{(0)}(p_f, p_i)$, they are not relevant to Gordon ambiguity and so are not considered in the present work. In deriving Eq. (31), only the general relations, the Dyson equation (22) and the identities (24) and (25), have been used. Therefore, the above procedure can be applied to any higher-order corrections including charged-meson current, and so we can take the CC1, CC2 and CC3 current for $\Gamma^\mu_{(0)}(p_f, p_i)$ and Eq. (31) is valid for the complete expression of the nucleon current in a nuclear medium within the meson field theory.

Then we calculate the proper current in nuclear medium from the CC1 current for $\Gamma^\mu_{(0)}(p_f, p_i)$. For the purpose, it is convenient to take the contraction of the current with q_μ. Using

$$\left[\,\Delta_i(p_i) + \Delta_f(p_f)\right]^n q^\mu = 0, \tag{32}$$

we obtain

$$q_\mu\, \Gamma^\mu(p_f, p_i) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\,\Delta_i(p_i) + \Delta_f(p_f)\right]^n q_\mu\, \Gamma^\mu_{CC1}(p_f, p_i). \tag{33}$$

Substituting

$$q_\mu \Gamma^\mu_{CC1}(p_f,p_i) = F_1(q^2)\left[\left(\not p_f - M\right) - \left(\not p_i - M\right)\right] - \frac{1}{2M}F_2(q^2)\left[\left(\not p_f - M\right)^2 - \left(\not p_i - M\right)^2\right], \tag{34}$$

we have

$$q_\mu \Gamma^\mu(p_f,p_i) = q_\mu \Gamma^\mu_{CC1}(p_f,p_i) + \Pi_1(p_f,p_i) + \Pi_2(p_f,p_i) + \Pi_3(p_f,p_i), \tag{35}$$

where

$$\Pi_1(p_f,p_i) = F_1(q^2)\left[\Delta_i(p_i) + \Delta_f(p_f)\right]\left[\left(\not p_f - M\right) - \left(\not p_i - M\right)\right], \tag{36}$$

$$= -F_1(q^2)\left(U_f - U_i\right), \tag{37}$$

$$\Pi_2(p_f,p_i) = -\frac{1}{2M}F_2(q^2)\left[\Delta_i(p_i) + \Delta_f(p_f)\right]\left[\left(\not p_f - M\right)^2 - \left(\not p_i - M\right)^2\right], \tag{38}$$

$$= \frac{1}{2M}F_2(q^2)\left(U_f \not q + \not q U_i\right) + \frac{1}{2M}F_2(q^2)\left[\left(\not p_f - M\right)(U_f - U_i) + (U_f - U_i)\left(\not p_i - M\right)\right], \tag{39}$$

$$\Pi_3(p_f,p_i) = -\frac{1}{4M}F_2(q^2)\left[\Delta_i(p_i) + \Delta_f(p_f)\right]^2\left[\left(\not p_f - M\right)^2 - \left(\not p_i - M\right)^2\right], \tag{40}$$

$$= -\frac{1}{2M}F_2(q^2)\left(U_f^2 - U_i^2\right). \tag{41}$$

Here it is assumed that $\Delta_{i(f)}(p_{i(f)})$ does not operate on the phenomenological form factors $F_{1(2)}(q^2)$.

Using the Dirac equation for the initial and final (renormalized positive-energy) state wave-function,

$$\left(\not p_i - M - U_i\right)\psi_i = 0, \tag{42}$$

$$\bar\psi_f\left(\not p_f - M - U_f\right) = 0, \tag{43}$$

the second term of Eq. (39) cancels out Eq. (41). Therefore we have the current expression as

$$\Gamma^\mu(p_f,p_i) = \Gamma^\mu_{CC1}(p_f,p_i) + \frac{1}{2M}F_2(q^2)\left(U_f\gamma^\mu + \gamma^\mu U_i\right) - F_1(q^2)\left(U_f - U_i\right)\frac{q^\mu}{q^2}. \tag{44}$$

The second and third terms are the corrections to the CC1 current. Substituting Eq. (6) and using Eqs. (42) and (43) again, we can see that the second term of Eq. (44), which corresponds to the vertex renormalization of Eq. (19), offsets the second term of Eq. (6), which corresponds to the wave function renormalization of Eq. (16). As a result Eq. (44) becomes

$$\Gamma^{\mu}\left(p_f, p_i\right) = \Gamma^{\mu}_{CC2}\left(p_f, p_i\right) - \left(q^{\mu}/q^2\right)\left[\, q \cdot \Gamma_{CC2}\left(p_f, p_i\right)\right]. \tag{45}$$

The second term is just the Landau gauge prescription [1,11] for restoring current conservation when the CC2 form is used in nuclear medium. However the proper current Γ^{μ} has such a term naturally. There are no needs to include it by hand.

In the practical calculations of physical quantities, the second term of Eq. (45) has no effects [1,11]. Therefore we recover the CC2 current. In fact, if the proper current is calculated from the CC2 for $\Gamma^{\mu}_{(0)}\left(p_f, p_i\right)$ in Eq. (31), Π_2 and Π_3 in Eq. (35) disappear and only the Π_1, which produces the second term of Eq. (45), remains. In the recent full DWIA analysis of (γ, n) reaction [8], the CC1 current can reproduce experimental data fairly well, while the CC2 current failed. As the authors pointed out, this success of the CC1 current is spurious due to the serious effect of two-body mechanism as meson exchange current on the (γ, n) reaction [12].

Because the above calculation is quite formal and general, we can also calculate the proper current from the CC3 for $\Gamma^{\mu}_{(0)}\left(p_f, p_i\right)$ by using

$$\begin{aligned} q_{\mu}\Gamma^{\mu}_{CC3}\left(p_f, p_i\right) &= F_1\left(q^2\right)\left[\left(\not p_f - M\right) - \left(\not p_i - M\right)\right] \\ &\quad + \frac{1}{2M}F_1\left(q^2\right)\left[\left(\not p_f - M\right)^2 - \left(\not p_i - M\right)^2\right] \end{aligned} \tag{46}$$

in place of Eq. (34). In this case $F_2(q^2)$ in Eqs. (39) and (41) are replaced by $-F_1(q^2)$. Thus Eq. (44) becomes

$$\Gamma^{\mu}\left(p_f, p_i\right) = \Gamma^{\mu}_{CC3}\left(p_f, p_i\right) - \frac{1}{2M}F_1\left(q^2\right)\left(U_f\,\gamma^{\mu} + \gamma^{\mu}\,U_i\right) - F_1\left(q^2\right)\left(U_f - U_i\right)\frac{q^{\mu}}{q^2}. \tag{47}$$

Using Eqs. (42) and (43), the second term of Eq. (47) offsets the second term of Eq. (7). Consequently, the result (45) is also obtained. It is not strange to obtain the same result for both CC1 and CC3 current. It is essentially due to the fact that the Ward identity itself does not depend on the value of ξ in Eq. (12).

We have studied the Gordon ambiguity of the nucleon current in nuclear medium described by the relativistic mean-field model. First, for the isoscalar current at $q = 0$, it has been shown that the difference between the CC1 and CC2 current is just the effect of wave function renormalization, but is cancelled out by the effect of vertex renormalization due to the Ward identity. Next the investigation is generalized to full CC1 and CC3 current. We have explicitly extracted the effects of mean-fields on the current by the terms of $n \geq 1$ in Eq. (31), which is valid for the complete vertex in nuclear medium. Then we have calculated the proper current and obtained the unified result of Eq. (45) whether the CC1, CC2 or CC3 is adopted for $\Gamma^{\mu}_{(0)}(p_f, p_i)$. Important point is a consistent treatment of the internal and external propagators of a nucleon in Feynman diagram. In conclusion the proper current is never worried about the Gordon ambiguity and the familiar CC2 current is reasonable. This naturally explains the results of recent fully RDWIA analyses of $(e, e'p)$ [7] and (γ, p) [8] reactions.

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